UNIVERSITY OF ALBERTA

LISTENING TO REASON:
AN INQUIRY INTO MATHEMATICS TEACHING

BY

BRENT A. DAVIS

©

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This work is dedicated to my parents,
Ruby Kathleen Davis and **Gordon Robert Davis**.

•

I really was listening.

ABSTRACT

This document is a speculative essay involving the analysis and the re-interpretation of several critical issues surrounding the practice of teaching mathematics —including the nature of mathematical knowledge, the place of education, and the processes of learning. The essay is developed around the notion of "mathematics teaching as listening," a phrase which is used both figuratively and literally: both as a metaphoric lens to re-interpret various phenomena and as a practical basis of teaching action.

The discussion of each issue begins with the broad ("theoretical") considerations and moves to the more specific ("practical") implications for teaching. This structure is intended to reflect the underlying theoretical (enactivist) and investigative (hermeneutic) frameworks which—drawn from recent developments in such fields as Continental philosophy, ecology, biology, and cognitive science—are rooted in post-Darwinian evolutionary metaphors rather than the analytic model of Descartes. As such, they help us to sidestep the sorts of dualistic thinking that give shape to much of current mathematics teaching. In their place are offered conceptions of human action, identity, and agency as fluid, dialogical, complex, and beyond our attempts to pre-determine. These points are further developed in a phenomenology of listening, in which it is suggested that listening is an imaginative and interactive participation in the formation and the transformation of the world (versus a "taking in" or a solitary construction of a world) which involves a continuous interrogation of one's perceptions.

The topic of the second discussion is the nature of mathematics. An alternative to the current "realist versus fallibilist" debates is sought through a history of the discipline. I conclude that mathematical knowledge is better understood in terms of *hearing* and *sounds* rather than the currently preferred *seeing* and *objects of vision*. The curricular and lesson-planning implications of this alternative are also explored.

The focus of the third discussion is formal education. There I present and critique a range of prominent perspectives, endeavoring to open a space for an enactivist alternative that allows us to more deeply understand the reciprocal effects of our society and our systems of education. The teacher, in this conception, cannot be detached and neutral, but is fully implicated in the subject matter. An awareness of the pedagogical relationship between teacher and learner—a relationship founded on listening—becomes critical to the teaching of mathematics.

The fourth discussion focuses on knowing, learning, and understanding. Radical constructivist and enactivist alternatives to conventional conceptions are presented. The role of "play" (used polysemously) in learning is investigated.

The strands of these discussions are brought together in a final discussion of teaching. The sonorous basis of mathematical knowledge, the hermeneutic structure of the educational endeavor, and the dialogical nature of our understandings contribute to a teaching founded on our capacities to listen.

ACKNOWLEDGMENTS

The term "complexification" is a recent addition to the English language. I find the word appealing for two reasons: First, it points to what I perceive to be a general movement across a range of academic disciplines—away from attempts to impose linear and causal models onto phenomena and toward embracing the difficulty and ambiguity of existence. Second, it is a new word—a new pattern of acting. Its creation represents a deliberate attempt to affect the way we stand in the world. As Richard Rorty argues, it is by making up new words (or by using old ones in new ways) that we interrupt the *common sense* notions which frame our actions. We enable ourselves to act differently.

Such is the spirit of this study: it is an attempt to complexify the phenomenon that we call "mathematics teaching" through suggesting a different way of speaking about it.

One of the first things the complexifier realizes is that most phenomena are not only more complex than we might have been led to believe, they are tied up in one other in ways that belie unraveling. Nothing is isolated; each thing affects every other thing. The task, then, of identifying those persons who and those events which contributed most significantly to the outcome that is this dissertation is something of a guessing game. I therefore apologize to anyone who is not appropriately acknowledged below. I readily confess that there are many persons whose contribution to this particular piece of work has been undeservedly overlooked. But there are those to whom I can point in confidence.

To begin, Tom Kieren's presence can be read on every page. Exemplary pedagogue and outstanding scholar, I can pay no greater respect than to describe him a true, fully-embodied listener. I am indebted to him and to the other members of my examining committee for their generosity, their confidence, and their encouragement. Great debts are also owed to many of my fellow graduate students. Their depth of concern, the strength of their character(s), and the extent of their talent have made it an honor to have been counted among them.

If you read much of this dissertation, you will notice that I enjoy trying to bring together varied influences and diverse strands of thought. I attempt to live my life in the same way, blending professional with personal, academic with recreational, social with private. Some boundaries have become so blurred that I am not often able to distinguish my own actions from the actions of those nearest me. It is thus that I find myself unable to pinpoint the contributions of Dennis Sumara to this writing. Those contributions have nonetheless been immense. I thank him with all my heart.

Finally, I express my appreciation for the financial support provided by the Social Sciences and Humanities Research Council of Canada, by the University of Alberta, and by the Grande Prairie School District #2357.

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Front Word

SETTING THE TONE
Introduction

The ironist's preferred form of argument is dialectical in the sense that she takes the unit of persuasion to be a vocabulary rather than a proposition. Her method is redescription rather than inference. Ironists specialize in redescribing ranges of objects or events in partially neologistic jargon, in the hopes that by the time she is finished using old words in new senses, not to mention introducing brand new words, people will no longer ask questions phrased in the old words. So the ironist thinks of logic as ancillary to dialectic.[1]

The opposite of irony is common sense.[2]

—Richard Rorty

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[1] Richard Rorty, *Contingency, Irony, and Solidarity* (New York: Cambridge University Press, 1989), 78.
[2] Ibid., 74.

Setting the Tone

It is common practice, before embarking on the recital of a long journey,
to give one's listeners an overview of the territory to be covered.
This is especially valuable when the itinerary involves twists and turns, detours, and steps retraced.
It makes it possible, at any moment to get a feeling for the overall picture.
—Hubert Reeves[3]

From the Visual to the Auditory

"Sound," wrote Erwin Strauss, "is something *between* thing and nothing. . . . [Sound] *is* something, yet it is not a thing one can manipulate; . . . it is not a thing, but neither is it no-thing."[4]

Sound does not allow itself to be held. Its transience, its impermanence, its instability, its insubstantialness make it impossible to be grasped by our listening in the way objects of vision are halted, distinguished, and possessed by our sight. Sound reminds us of the temporality of every part of our existence.

But we live in an era that seeks constancy, uniformity, totality, clarity, and distinctness,[5] and so, while the visual is valued, the auditory tends to be held in contempt: *Seeing is believing*, but *you can't believe everything you hear.* Nowhere is this mind-set more audible than in the discourse that surrounds the teaching of mathematics. Education has become a matter of *enlightenment*, where understandings are *insights* and teaching is a process of *illumination*. The *ideas* and *theorems*[6] of mathematics, situated at the core of a modern education, have come to be *seen* as the epitome of fixed knowledge. In Western societies, the discipline is afforded the status of pure reason, and the popular belief is that mathematics provides us with a *lens* to *uncover* the hidden, to *clarify* the obscure, to *revise* the mistaken, and to *expose* the false.

The underlying premise of this document is that our desire for clearer vision—and for the absolutes that it promises—has brought on a sort of cultural deafness. Notions of harmony and attunement have been pushed into the realms of the quaint and the romantic in a quest for monotonic truth. Learners—those we are to teach—have been reduced to silence; they are objects to be *seen and not heard.* My basic argument is thus that there is much to be gained by exploring sonorous alternatives to the visual metaphors that frame our teaching of mathematics. Sound—in its multi-layered richness, in its capacity for formless confusion, in its necessary proximity, in its inevitable transience—provides us with better *senses* of knowledge, of human interaction, and of personal identity, for, as Strauss has announced, it places us in the fractional dimension between thing-ness and no-thing-ness. Listening, the sound sense, might thus be said to occur in this undefined space, somewhere between the surety of the known and the uncertainty of the not-yet-known. It is, after all, when we are not certain that we are compelled to listen. Our

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[3] Hubert Reeves, *Malicorne: Earthly Reflections of an Astrophysicist* (Toronto, ON: Stoddart, 1993), 16.

[4] Erwin Strauss, *Phenomenological Psychology* (New York: Basic Books, 1966), 286.

[5] This list is drawn from David Michael Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics* (London: Routledge, 1989), 31. He uses these terms and others to describe what he call a "metaphysics of vision"—a modern technocratic mind-set that he argues to be pervasive in Western cultures.

[6] Both these terms are derived from words that had to do with seeing and vision.

listening is always and already in the transformative space of learning. It thus offers a rich playground in which to re-explore and to re-interpret teaching.

The Setting

There is a certain debate-like quality to much of the current discussion in the field of mathematics education. Theorists, researchers, and teachers alike are struggling to understand the implications of "new" and "radical" theories of cognition by which learning is recast as a dynamic and individual process of constructing, rather than acquiring, knowledge.

Although there are some critical differences among the current theoretical offerings, those espousing them tend to gather under the banner of "constructivism," seeing as their task the displacement of those culturally-privileged and historically-entrenched perspectives and practices that have, until only recently, gone virtually unchallenged. As far as the theoretical foundations of the field go, the impact of the constructivist movement has been nothing short of revolutionary. However, for reasons that I explore later, the dramatic theoretical shifts have had virtually no impact on the actual teaching of mathematics.

More recently, innovative perspectives on such issues as the nature of mathematical knowledge and the role of formal education in our culture have posed serious challenges to many of the long-held beliefs undergirding mathematics teaching. For the purposes of this introduction, the challenges that we currently face might be expressed as a series of as-yet unresolved tensions. Regarding mathematics, for example, we are faced with the question of whether mathematical "objects" have a prior, real existence or whether the "truth" of mathematics is a matter of intersubjective agreement. On the issue of education, theorists from a range of perspectives are questioning the purposes and consequences of formal schooling, arguing that the educational system has become a mechanism for suppressing rather than enabling the persons that it is ostensibly designed to serve.

I have developed this document around an investigation of these three issues (i.e., the processes of learning, the nature of mathematical knowledge, and the role of education), guided by Rorty's admonition:

> We can only hope to transcend our acculturation if our culture contains . . . splits which supply toeholds for new initiatives. Without such splits—without tensions which make people *listen* to new ideas in the hope of finding means of overcoming those tensions—there is no such hope.[7]

I thus do not endeavor to resolve the current tensions, but to explore alternatives to the system of thought on which they are founded. I use the notion of *listening* both as a starting place for the project and as a sort of collecting point for the various strands of thought that challenge our modern ways of *seeing* things—ways which I argue to be divisive and violent. Listening suggests a more generous, more compassionate, more encompassing alternative: whereas we *steal* a glance and *take* a look, we *lend* an ear and *give* a listen.

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[7] Richard Rorty, *Objectivity, Relativism, and Truth: Philosophical Papers, Volume 1* (New York: Cambridge University Press, 1991), 13-14. (emphasis added)

Further to this point, the modern vision-based attitude has been described by ecological theorist Wendell Berry as "specialization." For him, specialization is "the disease of the modern character."[8]

> What happens under the rule of specialization is that . . . [the] community disintegrates because it loses the necessary understandings, forms, and enactments of the relations among materials and processes, principles and actions, ideals and realities, past and present, present and future, men and women, body and spirit, city and country, civilization and wilderness, growth and decay, life and death—just as the individual character loses the sense of a responsible involvement in these relations.[9]

I am in accord with Berry, and my central thesis—that, as educators, we must become better listeners—is in direct response to his concern. Listening does not specialize; it may be in fact a cure for specialization. A juxtaposition of David Michael Levin's description of listening with Berry's critique of specialization makes this contrast clear:

> [A] better, more developed competence in listening could significantly improve the communicative infrastructures of the lifeworld that are necessary conditions for rational consensus, legitimation, equity, and justice. . . . [The] cultivation of this capacity can contribute to and is in turn affected by, the forming of moral character, encouraging communicative relationships, awakening a compassionate sensibility and the understanding it bears within it, motivating a concern for reciprocity and respect for differences, enabling the recognition of authentic needs, reversing processes of alienation that disintegrate the Self, and transforming the patriarchal ego, the historically constellated source of a will to power that has turned itself more and more into an instrument of nihilism, raging self-destructively.[10]

Listening, as conceived by Levin and as developed herein, then, involves far more than the "taking in" of sound. Listening, rather, is more toward an imaginative participation in the unfolding of the world. Immediate, intimate, and interactive, listening is more an interrogation of one's perceptions than the mere sensory capacity.

What might this study be?

This is an inquiry into the teaching of mathematics. While it has been oriented by an issue (i.e., the role of listening) rather than a specific question, the essence of the project, I believe, is best suggested by the query, What *might* mathematics teaching be?

That is to say, in this research I have not sought to determine, once and for all, what mathematics teaching *is*. Such an endeavor, I feel, would be futile, for there is surprisingly little consensus on such "straightforward" topics as the nature of mathematics, the role of education, and the processes of learning—let alone the issue that lies at their point of intersection: the teaching of mathematics. Further, attempts to determine and fix the "truth"—the *is*-ness—of phenomena in this modern age, tend to

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[8] Wendell Berry, *The Unsettling of America: Culture and Agriculture* (San Francisco: Sierra Club Books, 1977), 19.
[9] Ibid., 21.
[10] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 21.

have a sinister and hidden agenda. It often seems that the driving rationale behind the desire to know what something *is* is to secure some sort of mastery over it. The purpose of knowing what mathematics is, for example, is to subject it to our control; education we seek to manage; cognition we strive to colonize; teaching we endeavor to technologize. In effect, by seeking the *is*-ness of a phenomenon, we are announcing our intention to create an object of it.

But, by allowing the phenomenon to remain in the realm of *might*-ness, we allow it to remain, in the manner of sound, in the space between thing and no-thing.

Moreover, questions of "What is . . . ?" tend to focus strictly on the immediately present—the *now*. As such, their answers are often formulated in the jargon of conventional perspectives and entrenched practices. The question, "What might . . . ?," in contrast, announces a tentativeness, an openness—the possibility of other ways of thinking and being. It abandons the quest for reductive certainty and thus brings a hope for transformation. I have thus written not in terms of actuality, but of possibility and necessity—in Roger Scruton's words: "to what might be and what must be, rather than to what is."[11]

I have, in effect, undertaken to articulate another possibility for mathematics teaching—one that challenges the "taken-for-granted" of current conceptions. As such, this document cannot be understood as a research report. It is, rather, more along the line of a speculative essay. As William Schubert explains,

> [The essay] is a form of writing quite unlike the research report which summarizes the product of empirical inquiry. In contrast, the essay lets the reader travel the undulating trek of thought and feeling that the essayist travels. This, of course, means that every essay will be of a different method, in fact, the essay symbolizes the essayist in search of method.[12]

This attitude of open searching is implicit in the word "essay." Coined by the French from the verb *essayer* (to try), "essay" announces a spirit of *trying on* and *testing for fit*. The goal of the essayist is thus explorative—sounding out new patterns of living rather than seeking greater degrees of Truth.

In many ways, then, this dissertation is more play than display. It is a not a recounting of findings, but an active part of the search. In effect, this project represents a personal attempt to move from a conception of mathematics teaching as a specialization to a conception of mathematics teaching as a process of listening. Similarly, the research might more appropriately be described as a process of "listening" rather than in terms of established methodologies.

As might be expected, then, this project has demanded that I not specialize. Rather, it has compelled me to be eclectic and, as a result, relatively little of what is contained herein is directly founded on current research in mathematics education. The conceptual framework, instead, has been influenced by recent thought in the areas of continental philosophy, biology, ecology, curriculum theory, mathematics, and literature. The form of this dissertation is web-like, involving a weaving and re-weaving of strands of thought from these diverse areas. It is thus that, considered separately, none of the

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[11] Roger Scruton, *A Short History of Modern Philosophy: From Descartes to Wittgenstein* (London: Routledge, 1981), 7.

[12] William H. Schubert, "Philosophical Inquiry: The Speculative Essay," in *Forms of Curriculum Inquiry*, ed. Edmund Short (New York: SUNY Press, 1991), 65.

ideas presented is likely to provoke a different way of thinking about mathematics teaching. It is in the fabric of their interrelations that that possibility is presented.

The Structure of the Writing

An immediate consequence of the fact that this text is more an essay than a report is that a particular sort of reading is demanded. One cannot get much of a "feel" for what is written by going through this introduction, glancing over the gathered "data," and reading the final chapter. What conclusions there are have been distributed throughout the text and, rather than attempting to corral the educational implications into some sort of closing summary, I have conceived of this entire project as being educational. As a teacher, who is guided by a conviction that the place of the teacher is not simply to re-present others' ideas, but also to present possibilities, I have attempted to make this research educational.

I have thus endeavored to push against the linear limitation of written language by taking the single thread of text and weaving it in both vertical and horizontal directions. That is, I have given each chapter a parallel structure in an attempt to develop two sets of simultaneous dialogues—one of which moves through a series of issues surrounding mathematics teaching; the other of which involves an exploration of the theoretical foundations of this text at various organizational and organismic levels. The issues that serve as gathering points for the discussion (and as the themes for the chapters) are: the nature of mathematical knowledge, the place of formal education, and the processes of cognition. These intertwining topics emerge from and contribute to our conceptions of mathematics teaching. Similarly, the layers of the discussions contained in each chapter range from the global cultural/collective, through the institutional/interpersonal, to the more particular and immediate personal/individual considerations that are relevant to any discussion of mathematics teaching.

The text is thus divided into five chapters, and each of these chapters is comprised of three sections. Schematically, the structure might be represented by a rectangular grid in which the issues and levels are identified, respectively, on vertical and horizontal axes. (See Table 1, on which key words from each of the resulting "cells" are also identified.)

SECTION → / ↓ CHAPTER	A. Cultural	B. Institutional	C. (Inter)Personal
1. **Conceptual Underpinnings**	Enactivism	Hermeneutics	Listening
2. **Subject Matter**	Mathematical Knowledge	Curriculum	Planning for Teaching
3. **Formal Education**	Culture Building	The Artistry of Teaching	Pedagogy
4. **Cognition**	Knowledge and Knowing	Understanding and Meaning	Play
5. **Teaching**	Occasioning	Assessment	Mathematics Teaching as Listening

Table 1. A Conceptual Framework for the Structure of the Dissertation

The text might just as easily have been divided into three chapters, each with five sections—and I would, in fact, recommend for anyone who cares to go through the document a second time to follow a sequence of corresponding sections rather than successive chapters.

One of my reasons for developing this structure was to avoid having to provide extensive elaboration of isolated points. Some of the topics that are dealt with have been difficult for me—difficult not because they are conceptually complex, but because they tend to move against the current of *common sense*; they resist being framed in conventional terms. In writing, I was thus faced with either attempting to provide detailed explications of issues as they came up or with presenting pivotal ideas as I went along and postponing elaboration until need or opportunity arose. I chose the latter.

As such, I might characterize the structure as *layered*. Each chapter (and section) picks up on the ideas of the preceding chapters (and sections), thus adding to the conceptual depth. In this way, the document has taken on a sort of recursive structure the might be interpreted visually with a series of concurrent circles (see Figure 1). Each successive layer encompasses and expands on that which has preceded it, and, in this elaboration, effectively transforms the meaning and significance of what has come before. The process is therefore somewhat circular, multi-dimensional, and unending.

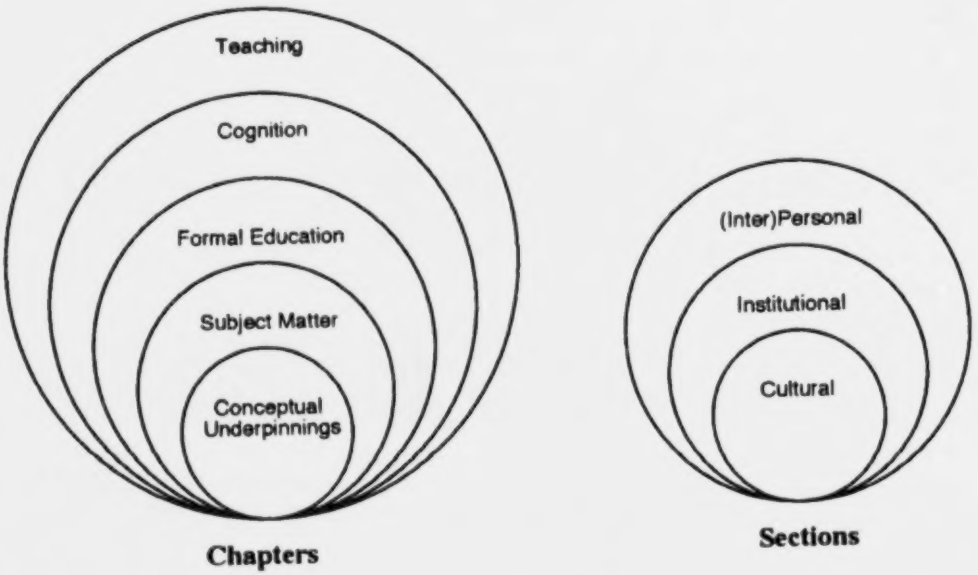

Figure 1. The Layered and Self-Similar Structures of the Chapters and Sections[13]

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[13] This structure is borrowed and adapted from Kieren and Pirie's model of mathematical understanding. See Susan Pirie and Thomas Kieren, "Growth in mathematical understanding: How can we characterize it?" in *Educational Studies in Mathematics* (forthcoming).

As these two sets of dialogues are laid atop one another (see Figure 2), a perhaps more representative (and certainly more descriptive) scheme than that suggested by a rectangular grid (as in Table 1) is created.

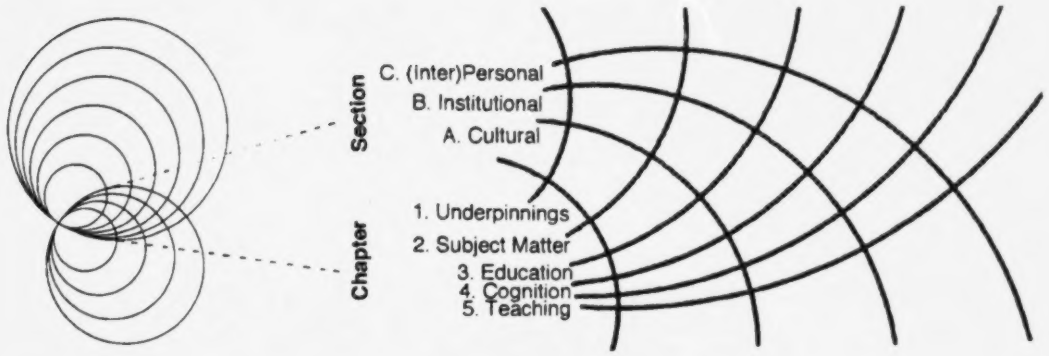

Figure 2. The Recursive Structure of the Dissertation

This layered approach highlights two features of this writing that reflect important aspects of the experienced world to which it points. The first is that the complexity at various levels is not a function of scale. Rather, as one moves in on or away from a phenomenon, one inevitably finds the same order of complexity. The second feature is revealed as the exploration moves between cultural, institutional, and personal layers of the discussion. A certain self-similarity emerges as the same issues, the same images, and the same metaphors seem to arise on very different topics and conceptual levels. The phenomenon to which I am referring has been succinctly characterized by physicist Richard Feynman: "Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry."[14] My project here is to tug at a few strands, endeavoring to arrive at a different sense of how, just as parts are spread throughout the whole, the whole is enfolded in all the parts.

As a sort of a reminder of this relationship between the general and the particular, I have adorned the title pages of each chapter of this dissertation with an image of a *fractal tree* at some stage of its growth. Briefly, a fractal tree is generated by repeatedly grafting a single shape—in this case, a simple two-pronged fork (see Figure 3), the tines of which are approximately two thirds the length of the handle—onto a larger image of itself. With each iteration, the tines of the preceding level become the handles of the new fork; one might say, in terms of a tree's growth, that last season's shoots become this year's branches.

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[14] Richard P. Feynman, *The Character of Physical Law* (Cambridge, MA: The MIT Press, 1965).

Figure 3. The Generator for the Fractal Tree.

The fork-shape thus serves as a "fundamental particle" of sorts--the basic building block or the generator—of the tree. The information required to construct the entire figure is contained in each tiny fractal element, although the complexity of the "completed" figure may obscure the simplicity of its generative subunits. The "fractal" thus differs from the "fragment" in that it not only contributes to the whole, it announces it. However, in this simplicity, it belies the complex and unpredictable patterns that begin to appear as the iterated and re-iterated elements start to play on themselves.[15]

The introductory understanding of fractal geometry that enabled the drawing of this figure has profoundly affected the way I see trees. It has also affected the way I understand a host of other phenomena, including the emergence of a new idea. It is thus that I have attempted to imbue each of the fifteen subsections of this document with a certain fractal-like integrity. That is, rather than permitting each piece to serve merely as a fragment of the whole—something virtually meaningless on its own—I have striven to hint at the greater richness of the more fully articulated document by exploring in-depth a specific topic in each piece.

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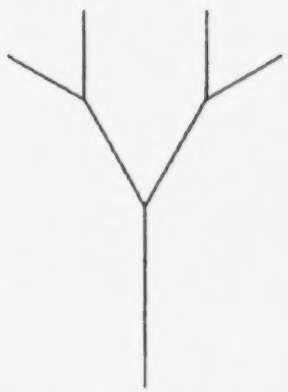
[15] For the purposes of illustration, the reader might compare the simple fork with the completed tree (i.e., "completed" in the sense that the capacity of the drawing program was extended to its limit), as presented on the title page of the bibliography. There, circles, spirals, webs, and polygons of all sorts emerge and contribute to a rather dense "foliage."

Chapter 1

CLOSE YOUR EYES AND LISTEN

Conceptual Underpinnings



Just as nature finds its way to the core of my personal life and becomes inextricably linked with it, so behavior patterns settle into that nature, being deposited in the form of a cultural world. Not only have I a physical world, not only do I live in the midst of earth, air and water, I have around me roads, plantations, villages, streets, churches, implements, a bell, a spoon, a pipe. Each of these objects is molded to the human action which it serves. Each one spreads round it an atmosphere of humanity.

—Maurice Merleau-Ponty[1]

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[1] Maurice Merleau-Ponty, *Phenomenology of Perception* (London: Routledge, 1962), 347.

Section A

Enactivism

> [O]rganism and environment enfold into each other
> and unfold from one another
> in the fundamental circularity that is life itself.
> —Varela, Thompson, & Rosch[2]

Much of the activity in the field of mathematics education consists in efforts to negotiate a series of impassable dichotomies—dichotomies which seem to be the direct and inevitable consequences of the collision between traditional objectivist perspectives and more recent subjectivist proposals. In this section, I examine the origins of the mode of bi-polar thinking that has given rise to these tensions, seeking not to resolve them but to understand them more deeply. It is by endeavoring to develop such understandings, I suggest, that we open up the possibilities for not merely closing the "gaps," but for side-stepping the mode of thinking (and acting) out of which they arise.

Put differently, I begin by suggesting the dichotomies are false ones, tied to our inability to escape a modern mind-set which posits us as essentially autonomous entities: not only are we isolated from one another, we are set apart from the universe. The foundation of this sort of dualistic thought is the topic I turn to presently. That discussion serves as a precursor to a brief introduction of an alternative orientation to issues of identity and cognition which, for the moment, I will describe as embracing complex and unpredictable evolutionary dynamics rather than imposing orderly and calculable mechanical processes.

Our Modern Heritage

The predominant epistemological perspective of the "modern" era was first announced by mathematician and philosopher René Descartes in the seventeenth century. Descartes, a contemporary of Galileo, Bacon, and Locke, and a predecessor of Newton, articulated two breaks from earlier perspectives on knowledge and modes of inquiry—perspectives which he rejected as inconsistent and unreliable mixtures of fact and fancy.

The first point of departure was on the issue of method, whereby Descartes denounced tradition, hearsay, mysticism and religion as he called for the pre-eminence of the "natural light" of (mathematical) reason. Voicing a disdain for all other intellectual authorities, Descartes argued that all previous speculation should be rejected until indubitable principles, against which all other knowledge claims could be measured, were derived. In calling for this shift to a particular and narrow conception of *reason*, Descartes introduced many concepts and arguments which are foundational to modern science and analytic philosophy.

In this regard, perhaps his most noted contribution is his *cogito*—"I think"—which also marks his second break with tradition. Briefly, in his quest for a certain foundation for his epistemological system, Descartes arrived at the self-evident and self-

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[2] Francisco Varela, Evan Thompson, & Eleanor Rosch, *The Embodied Mind* (Cambridge: The MIT Press, 1991), 218.

verifying truth of the statement, "I think," and this axiom became the solid ground on which he sought to verify or refute all other knowledge.

It is important to note that Descartes' project was built on a distrust of the evidence of the senses—a suspicion that was inherited from the ancient Greeks.[3] Because one's knowledge of the world was always and inevitably filtered through untrustworthy sensory organs, one could never know in any absolute way the "truth" of the (external) universe. At best, one could build better and better mental representations of the physical world, and the process of assembling these representations demanded a persistent attitude of questioning—an attitude that Descartes introduced as foundational to scientific inquiry. This "method of doubt" was offered as a screen to sift out *truths* from those knowledge claims that could not be validated. Rational thought was thus offered as a way of knowing that was both superior to and independent of a reliance on the senses. Descartes' model of reason—and the one that was to become *the* model of rationality in the modern world—was found in geometry, a discipline which offered a process of verification that Descartes regarded as the only route to unimpeachable fact.[4] For him, geometric reasoning offered a means of deducing the nature of the entire universe from foundational principles, with each deductive step bound to preceding steps in an irrefutable sequence of logical moves. It was thus that, according to Palmer, truth for Descartes was "more than merely the conformity between knower and known, it [was] the *subject's rational certainty* of this conformity."[5] Rational reflection ("rationalism") rather than empirical observation ("empiricism") was the key to knowledge.

In establishing the *cogito* as the foundation and geometric reason as the means of construction, Descartes initiated a mode of dualistic thought that permeates modern perspectives of the universe. Positioning the radical subject (i.e., the modern ideal of "self" or identity as solitary, coherent, and independent of context) as the reference point for all that is known, for example, compelled him to propose the existence of at least one object—an Other—that was independent of himself and relative to whom he could situate himself as part of an objective world. Thus arose the fundamental subject-object dichotomy—a dichotomy which, paradoxically, also provided the impetus for the empiricist tradition which, contra Descartes' rationalist proposal, relies on observation and experiment as the basis of knowledge production.

Another split initiated by Descartes' thought was the apparent independent constitution of mind and body. In arguing that thinking is the basis for all truth, and hence of existence, Descartes was suggesting that a person essentially a thinking thing—one that is capable of conceiving of itself as existing without a body. Put differently, it is not essential in Descartes' formulation that we have a corporeal existence. Of course, this mind/body separation finds its roots in earlier philosophical and religious thought. However, in giving it a rational "scientific" basis, Descartes set the stage for a series of tensions that now, collectively, serve as a pervasive and resilient backdrop to much, and perhaps most, of Western academic thought.

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[3] Plato had already argued that the world is revealed to reason alone and not to ordinary sense perception. See Scruton, *A Short History of Modern Philosophy: From Descartes to Wittgenstein*, 14.

[4] It is interesting to note that, at the time, mathematics was not highly regarded. In fact, it was neither seen as an independent discipline nor as a means of deducing unquestionable truths, and was as closely associated with mystical endeavors as with scientific. I return to this issue in the first section of the next chapter.

[5] Richard E. Palmer, *Hermeneutics: Interpretation Theory in Schleiermacher, Dilthey, Heidegger and Gadamer* (Evanston, IL: Northwestern University Press, 1969), 144. (original emphasis)

To elaborate, in constructing the world on the foundation of the *cogito*, Descartes articulated more than the separations of mind from body, self from other, and representation from reality—all of which might be described as manifestations of a mind/body dualism. In addition to the essential distinction between mental and physical objects (with the consequent priority being assigned to the former), Descartes also contributed to the foundation of the a host of other dichotomies, including knower versus known, organism versus environment, human versus nature. Further, the Cartesian orientation contributed to a view of the Self as a unified coherent subject: an autonomous entity that is isolated from others, independently constituted, essentially static, and able to maintain its integrity through diverse experience.

Other consequences of this perspective, which I shall heretofore refer to as "modernist," included an empirical emphasis on the trustworthiness of methods used to develop knowledge (i.e., more accurate representations of reality). As such, method came to be seen in increasingly mechanical and technological terms; the universe, correspondingly, was reduced to a similarly technical form. Today, machine metaphors frame and reveal Western perspectives on the universe, the earth, nature, our bodies, and—ultimately, with the development of the computer—our minds. With this technical mind-set, the aim of inquiry has grown beyond the desire to better our understandings. The primary goal is now to control the objects of our inquiry. As Palmer elaborates, with the widespread acceptance of Descartes' conclusion that "the world has meaning only with respect to man," our relationship to the world is no longer cast in terms of open responsiveness, but in "restless efforts to master it."[6]

And, perhaps most significantly, with thought being afforded priority over *being* in Descartes' *cogito*, epistemic issues began to overshadow ontological concerns—a reversal that has had profound implications for our modern conceptions of both knowledge and education. I will return to a further exploration of the consequences of modernist philosophies in later chapters that deal more specifically with these topics.

Foundations of an Alternative

It is interesting to note the prevalence in research reports of the claim that particular methodologies or perspectives are "anti-dualist." Reacting to a philosophical backlash against Cartesian (modernist or analytic) bi-polar thinking, researchers and theorists are quick to point out that they have not succumbed to a "this-or-that" way of thinking. Nevertheless, within mathematics education at least, there seems to be an irresistible tendency to grant priority to one or the other of the "real" *known* (material or abstract) or the ideal *knower*—tendencies which, like the favoring of either empirical or rational modes of inquiry, find their origins in the same system of beliefs.

And so, in spite of their apparent diametric opposition, these modernist perspectives can quite easily be shown to be on the same rational loop—a loop which begins with the epistemic primacy of "I think." The consequences of dualistic ways of thinking, along with extensive critique of such thought, are offered by a group of thinkers that tend to be gathered under the title of "postmodernism" (although not always by their own choosing). Unfortunately, while postmodern discourses have offered valuable critiques of Descartes' legacy, it seems that one of the precepts of postmodernism—i.e., that the quest for new groundings is doomed to failure—has been profoundly misinterpreted as suggesting that we can say very little about anything. Not surprisingly,

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[6] Ibid., 144, 146.

this conclusion has prompted numerous and zealous attempts to destroy the foundations of existing structures—thus demonstrating the temporal and contextual nature of all knowledge—while offering in their place the unsteady (and unsatisfactory) ground of fallibilist, relativist, and individualist accounts of knowing.

The new challenge thus seems to be the development of alternatives which abandon such assumptions but which do not give in to the temptations of establishing a new and irrefutable foundation. In this writing, I would like to explore one possibility that seems to be emerging from some convergent streams of thought that flow from such diverse disciplines and discourse fields as continental and pragmatist philosophy,[7] cognitive psychology,[8] ecological thought,[9] and biology.[10] Growing numbers of theorists in these areas are starting from the evolutionary metaphors of Darwin rather than the analytic and reductionist model of Descartes. Their focus is thus on the dynamic interdependence of individual and environment, of knowledge and identity, and of self and other, rather than on their autonomous constitution. Variously referred to as "pragmatism," "enactivism," "(deep or social) ecology," these strands of thought join with postmodern discourses to offer a critique of modern dichotomous thinking. They also, however, point toward another way of thinking which, unlike modernism and many of its current critiques, remembers our past and embraces the complexity of existence. Enactivist[11] theorists thus offer descriptions of knowledge and communication and models of cognition and learning which are historical, situational, dynamic, intersubjective, and consensual. More importantly, perhaps, and in sharp contrast to the modernist foci, they acknowledge the centrality of the phenomenal and experiential rather than fetishing the formulated.

The remainder of this section consists of an introduction to a few of the important elements of enactivist theories. Because these points will be elaborated upon in subsequent sections and chapters as they are applied to various issues related to the teaching of mathematics, this introduction is deliberately brief.

Searching for a Middle Way

Maurice Merleau-Ponty, a pre-eminent post-war French philosopher, has provided us with a radical reinterpretation of Descartes' *cogito*. His goal in this project was to find an alternative to the bi-polar divisive way of thinking that dominates Western scholarly thought.

In *Phenomenology of Perception*, Merleau-Ponty rejects both rationalist and empiricist accounts of perception—the former because it focuses too narrowly on the

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7 For example: Merleau-Ponty, *Phenomenology of Perception*; Rorty, *Objectivity, Relativism, and Truth: Philosophical Papers, Volume 1*; Hans-Georg Gadamer, *Truth and Method* (New York: Continuum, 1990).

8 For example: Jean Piaget and Bärbel Inhelder, *The Psychology of the Child* (New York: Basic Books, 1969); L. S. Vygotsky, *Mind in Society: The Development of Higher Psychological Processes* (Cambridge: Harvard University Press, 1978); and Jerome Bruner, *Actual Minds, Possible Worlds* (Cambridge: Harvard University Press, 1986).

9 For example: Berry, *The Unsettling of America: Culture and Agriculture*; Gregory Bateson, *Mind and Nature* (New York: E. P. Dutton, 1979); James Lovelock, *Gaia, A New Look at Life on Earth* (New York: Oxford University Press, 1979).

10 For example: Humberto Maturana and Francisco Varela, *The Tree of Knowledge* (Boston: Shambala, 1987); Varela et al., *The Embodied Mind*.

11 Following Varela et al., I will be using "enactivism" as a gathering term to refer to the related schools of thought.

cognizing agent (thus failing to provide an adequate account of the "world"); the latter because it demands too great a correspondence between a real world object and the resulting perception. Seeking a middle ground between the mental and the physical (the inner and the outer), Merleau-Ponty suggests that the body is that which renders the mind and the world inseparable. Far from representing a discrete demarcation between subject and object, one's body is simultaneously of oneself and of the world. For Merleau-Ponty, then, the body is our means of belonging to our world—a world which shapes us and a world we participate in shaping.

Taking up this notion, Varela, Thompson, and Rosch (who bring together biology, continental philosophy, and Buddhist thought) have endeavored to elaborate upon Merleau-Ponty's "fundamental intuition of double embodiment."[12] In this conception, the body is understood both as an outer (physical-biological) and as an inner (lived experiential-phenomenological) structure. These structures are not opposed; rather we "continuously circulate back and forth between them."[13]

An understanding of their use of the term "structure" is critical here. Briefly, one's structure comes about from the combined influences of biological constitution and one's history of interaction in the world—a notion that recalls Vygotsky's contention that human identity is subject to the dialectical play between biology and history.[14] For the current purposes, a person's structure may be thought of as being loosely synonymous with her personality or her *self*. For the time being, however, I will be avoiding these terms because of the inflexibility and the permanence they connote when used in a modern context. One's structure, in contrast, is thought to be fluid, temporal and necessarily undergoing change. As Maturana and Varela put it, "Ongoing structural change of living beings . . . is occurring at every moment, continuously, in many ways at the same time. It is the throbbing of all life."[15] Unlike modern conceptions of identity whereby one's *self* is regarded as a product, then, one's structure is product, producer, and process.

A person's range of possible action is determined by his structure, and hence, in an interaction with another person, how he acts is not a function of the other person's actions (as is presumed in transmission models of communication and teaching), but a consequence of his own structural dynamic. As Maturana and Varela explain

> [The] perturbations of the environment do not determine what happens to the living being; rather it is the structure of the living being that determine what change occurs in it. This interaction is not instructive, for it does not determine what its effects are going to be. . . . [The] changes that result from the interaction between the living being and its environment are brought about by the disturbing agent but *determined by the structure of the disturbed system*.[16]

To the observer, however, it may appear that one person is functioning according to the directions given to him by another person. Nevertheless, it is more appropriate to think of the interaction as a choreography in which one influences, but can not determine, the other. Put differently, one does not "pick up information" from the environment; rather, one's structure specifies which environmental patterns will trigger action. Furthermore,

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12 Varela et al., *The Embodied Mind*, xvii.

13 Ibid., xv.

14 Lev S. Vygotsky, *Thought and Language* (Cambridge, MA: The MIT Press, 1962).

15 Maturana and Varela, *The Tree of Knowledge*, 100.

16 Ibid., 96.

these environmental patterns (or, in Maturana and Varela's terms, "perturbations") do not *cause* the person's actions. Rather, they present an occasion for the person to act according to her structure.

In such interactions, one's structure is necessarily affected, although not always visibly, and she thus emerges a "different person." The other person involved in the interaction is similarly affected, and so the two "co-emerge." Varela et al. use the term *co-emergence* to call attention to the manner in which organism and environment, self and other are "bound together in reciprocal specification and selection."[17] That is, the world's relationship to the organism is not merely uni-directional and constraining; the organism also initiates or contributes to the enactment of its environment. They *specify* one another. In this *mutual specification*, they *co-emerge*. The "subject," in this conception, is not and can not be considered as disembodied or as objectively separated from the world. Both are entwined in the "fundamental circularity" of existence.

The full import of this notion is found in Varela et al.'s interpretation of the word "embodied," which is used to highlight two points. First, as is more commonly acknowledged, "cognition depends on the kinds of experience that comes from having a body with various sensorimotor capacities."[18] On this point, Mark Johnson goes so far as to suggest that words and concepts are metaphorical extensions of originary bodily experiences.[19] The second, and perhaps more critical point is that our sensorimotor capacities are embedded in and continuously shaped by broad biological, social, and historical contexts. Our knowledge and our identities—our structures or our embodiments—are thus dependent on "being in a world that is inseparable from our bodies, our language, and our social history."[20]

A more naive form of these ideas is found in current debates on the relative influence of nature and nurture on personality, intelligence and other (ostensibly fixable) traits. Viewed through the lens of enactivist theory, these debates miss two essential points: First, they tend to separate biological from social or historical factors, thus implying that the contribution of "nature" is fixed from birth. The separation of the terms "nature" and "nurture" suggests that they can somehow be held distinct—that nurturance is not natural and that what is natural is singularly and automatically constituted. Second, the debates presume a passive cognizing agent who is shaped by various forces, but who plays no role in selecting or affecting those forces. Enactivist theory denies the possibility for these easy separations, arguing that such distinctions are both impossible and unnecessary.

With regard to the nature of the individual knower, Varela et al. suggest that the basis of cognition is not to be found in the Rationalist "I think" nor in the Empiricist "I observe"—both of which are founded on the premise of the detached knower—but in the enactivist "I act." Acting encompasses both thought and observation; acting presumes both actor (subject) and acted upon (object). In brief, acting demands re-unions of mind and body and subject and object. It is this notion of embodied action that allows us to bypass the extreme positions of cognition as either recovering what is outer or projecting what is inner without seeking recourse in the supernatural or in metaphysics. The upshot is that cognition cannot be a matter of internalizing or forming accurate representations of

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[17] Varela et al., *The Embodied Mind*, 174.

[18] Ibid., 172.

[19] Mark Johnson, *The Body in the Mind: The Bodily Basis of Meaning, Imagination, and Reason* (Chicago: University of Chicago Press, 1987).

[20] Varela et al., *The Embodied Mind*, 149.

things of the world. Rather, cognition is inseparable from and fundamental to perception and action. Perceptions guide actions; actions enable perceptions. This inseparability is expressed in Maturana and Varela's aphorism, *All doing is knowing, and all knowing is doing*,[21] and the term "enaction" is intended to remind us of the primacy of action in shaping our experiences, our perceptions, and our world.

The individual's cognition, in this conception, is analogous to the evolution of a species, whereby an idea or an action comes about not because it is "correct" or optimal, but because it is possible in the given context. In making this provocative association between cognition and evolution (and their use of this comparison does appear to be more than metaphorical), Varela et al. are also indicating that enactivist notions can be applied at levels other than the organismic. Stated otherwise, the "cognizing organism" of their discussion need not be a single self-contained and visibly-bounded unit. Although the time scale varies, the developmental processes of the species and the individual are, in essence, the same: the cognizing agent (i.e., species or person) is engaged in a perpetual process of adapting itself to a similarly dynamic and responsive environment.

In other words, there is a fractal quality to enactivist theory, as suggested by this self-similarity across the conceptual levels of organism and species. Varela et al. have noted some of the implications of applying the theory at two such levels; other theorists have discussed similar ideas in reference to very different phenomena—but phenomena, if we are to explore the further implications of enactivist theory, must be seen as relevant to discussions of learning and education. Of note are James Lovelock's[22] "Gaia hypothesis" in which the ideas are applied on a planetary scale and Varela's earlier work with Maturana[23] which focuses on the interdependent and subordinate elements comprising our own biological structures. In the chapters that follow, I will be applying and exploring the implications of enactivist theory at three levels—cultural, institutional, and (inter)personal—as I investigate mathematics, formal education, learning, and teaching.

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[21] Maturana and Varela, *The Tree of Knowledge*, 26.
[22] Lovelock, *Gaia, A New Look at Life on Earth.*
[23] Maturana and Varela, *The Tree of Knowledge*.

Section B

Hermeneutics

> We are, as Proust declared,
> perched on a pyramid of past life,
> and if we do not see this,
> it is because we are obsessed by objective thought.
> — Maurice Merleau-Ponty[24]

I recently went for a walk along a gravel path in the river valley. The trail had been one of my favorites in the summer when it was enclosed by trees and filled with life, but now, at the end of October, it had lost its vibrancy. The green denseness of life was gone and the busyness of squirrels and insects had ceased. All that was left were the skeletons of trees and drifts of decaying leaves. It was a place of colorlessness, of disarray, of *not* life. Where once I felt embraced, I now felt isolated and exposed.

Until I heard a sound: a soft rustling of leaves. I listened. More rustling. Then a bird. Two birds. Many birds, layered on crickets and piled on top of the leaves—the leaves that moved with something living.

A simple story, really. But for me, it was a moment that powerfully announced something about listening. Caught in a moment of modern angst—relying strictly on vision to guide me—I felt lonely and separated. But, in listening, I found myself again entangled in the complex web of existence.

But the event was not just a moment of listening, for I was reminded of more than the ecologies of being. I was also recalled to my status as a listener (and as a non-listener), to my awareness of what I know (and did not know), to my role as a learner (and educator). It was a moment of locating order in ambiguity, of finding integrity and harmony amid the chaotic. It was a particular event, but an event that in its particularity revealed the general. As an event of being, it altered the tone of every walk along that path that preceded it and every walk that would follow.

And it was an event that, at the instant of living through it, I felt might serve as a means of introducing a discussion of how I have approached this investigation of learning and teaching. It has been a study of moving back and forth between the particular and the general, investigating how the former reveals the latter, how the latter is contained in the former, and how a change in understanding of one affects the other. In effect, this study of listening has itself been an act of listening: of tuning, of becoming attuned.

Put differently, the research reported herein was a hermeneutic investigation.

Some Background to Hermeneutics

Hermeneutics is the art of interpretation. It is concerned with meaning, with understanding, and with application. More particularly, hermeneutics is concerned with investigating the conditions that make certain understandings possible.

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[24] Merleau-Ponty, *Phenomenology of Perception*, 393.

Hermeneutic research seeks to illuminate the moments at which we move to a greater understanding of our uniquely human situation as it is relationally shared with others in the world. It understands that there is no truth which is fixed once and for all, no method which can pre-determine the location of truth, no authority who can say the way things "really are." As Joel Weinsheimer suggests, a motto for hermeneutics might be that "truth keeps happening."[25]

It is a field with a rich and a long history—a history which, because it has been thoroughly developed elsewhere,[26] I will address only briefly. Etymologically, *hermeneutics* is derived from the name and character of the Greek god Hermes, the messenger of the gods. The term thus echoes with senses of revelation and of coming to more profound understandings of the previously perplexing or paradoxical.

With regard to its modern use, *hermeneutics* was originally a discipline of biblical interpretation, the goal of which was to excavate the *truth* of the sacred text. This task demanded not just an ability to translate or comprehend particular words, but a talent to locate the writings historically and contextually. As such, from its inception as a discipline, hermeneutics has been concerned with the evolution of ideas and meanings, and it has been aware of the situational nature of such understandings.

Hermeneutics, then, has historically been concerned with textual interpretation, and its evolution is thus perhaps most easily outlined by tracing the development of the term "text." Initially, to the hermeneuticist, the *text* was a sacred writing. Later the notion of *text* was expanded to encompass literary and legal documents. More recently, the hermeneutic *text* has come to be life itself—an evolutionary development that reveals the shift from an original epistemological focus (i.e., biblical and literary hermeneutics were concerned with uncovering the *truth* in written texts) to the more ontological concerns of recent branches of hermeneutic thought. ("Philosophical" and "radical" hermeneutics address questions of the meaning of human existence as revealed through the rich and varied texture of particular experiences.) Implicit in this shift of the meaning of "text" is Heidegger's conviction that Being and interpretation are inseparable. As Crusius phrases this idea, "interpretation . . . is human being, our mode of existence in the world."[27]

An etymology of "text" is helpful here, in part because the term is conventionally understood to refer strictly to written works. Originally, however, "text," like "web," was used to describe things woven, and so the metaphor of "life as text" does have a particular richness. Considered alongside the more popular "literature as text" metaphor, for example, the image of intertwined linguistic threads forming a tightly woven narrative fabric foregrounds the roles of language, of story-telling, and of re-reading in the construction of our respective understandings and identities. The textual metaphor also offers an image of the interweaving of our selves in the fabric of our culture. Like the microscope and the telescope, which demonstrate that complexity is not a function of scale, the metaphor of text applied at different conceptual planes reveals that the web of existence is as tangled at the individual level as it is on a planetary scale. And, perhaps more importantly, the references to text, textuality, and texture remind us that we ourselves are woven into the fabric that we seek to understand. It is from inside our

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[25] Joel Weinsheimer, *Gadamer's Hermeneutics: A Reading of "Truth and Method"* (New Haven, CT: Yale University Press, 1985), 9.

[26] See, for example Palmer, *Hermeneutics: Interpretation Theory in Schleiermacher, Dilthey, Heidegger and Gadamer*; Gadamer, *Truth and Method*; Shaun Gallagher, *Hermeneutics and Education* (Albany, NY: SUNY Press, 1992).

[27] Timothy W. Crusius, *A Teacher's Introduction to Philosophical Hermeneutics* (Urbana, IL: National Council of Teachers of English, 1991), 5.

traditions that we interrogate them. We cannot view them from the outside or above; there can be no metaphysical (i.e., above or beyond the experiential) truth. Truth, from within this web, is more something to be listened for then something be looked at.

We are, then, caught up in "a finite existence prestructured by the tremendous inertial force of the past,"[28] an existence that is, in effect, pre-interpreted for us. The promise of hermeneutics is not to unburden ourselves of this historical mass in a (modernist) quest to determine *the* one Truth; nor is its goal, through more profound understandings of the world, to control the future or to better manage the objects that surround us. Rather, the place of hermeneutics is to interrupt our unquestioned patterns of acting. "All philosophical hermeneutics can hope to do . . . is hold open an alternative, constantly pointing to ways of living and thinking less destructive of the earth and the human spirit."[29]

To engage in hermeneutics—to interpret—then, is to tug at the threads of this existential text, realizing that, in tugging, the texture of the entire fabric is altered. Put differently, hermeneutics does not reduce us to powerless victims of historical forces. Rather, it offers hope for the future in the recognition that our lives our shaped not just by the events of the past, but also by our projects and our projections. Varela et al. refer to this phenomenon as the "fundamental circularity" of existence. In their words:

> We did not design our world. We simply found ourselves in it; we awoke both to ourselves and to the world we inhabit. We come to reflect on that world as we grow and live. We reflect on a world that is not made, but found, and yet it is also our structure that enables us to reflect upon this world. Thus, in reflection *we find ourselves in a circle*: we are in a world that seems to be there before reflection begins, but that world is not separate from us.[30]

Varela et al. are using the notion of "fundamental circularity" to illustrate that, while we are "thrown" (to borrow Heidegger's term) into a world that is not of our own making, once located, that world evolves with us. An event as mundane as a shifting thought, then, alters the whole universe for that thought—that thread—like the thinker and the context in which it occurred, is part of the universe.

The Hermeneutic Circle

David Smith notes that, beginning with the contribution of Schleiermacher, the three common themes in hermeneutic inquiry have been "the inherent creativity of interpretation, the pivotal role of language in human understanding, and the interplay of part and whole in the process of interpretation."[31] This third theme, the back-and-forth movement between the particular and general, is more popularly referred to as the "hermeneutic circle." As one moves between the specific and the broad, one's understandings of both are deepened, and all other understandings are also affected.

The embeddedness of the particular in the general, and the enactment of the general through the particular was first articulated by Schleiermacher who stated, "every

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[28] Ibid. 14.

[29] Ibid.

[30] Varela et al., *The Embodied Mind*, 3. (emphasis added)

[31] David G. Smith, "Hermeneutic Inquiry: The Hermeneutic Imagination and the Pedagogic Text," in *Forms of Curriculum Inquiry*, ed. Edmund Short (New York: SUNY Press, 1991), 190.

discourse depends on earlier thought . . . [and] it follows that every person is on the one hand a locus in which a given language is formed after an individual fashion, and on the other, a speaker who is only able to understand within the totality of language."[32] More grossly stated, in the study of interpretation, one's focus cannot be fixed on either the narrow or the broad, for fixing on an extreme denies the dialogic complexity of their fundamental circularity.

The notion is not unrelated to the "fundamental circularity of existence" as expressed by Varela et al. (noted above). Gadamer articulates a similar notion in his extension of the concept of the hermeneutic circle in which he more explicitly brings in the interpretive consciousness. For him, an interpretation involves first an appropriation of an event and, as one comes to meaning (interprets), a transformation of that event. The hermeneutic inquirer, in contrast to the social scientist,[33] thus cannot attempt to maintain the attitude of a detached observer whose goal is to provide an objective account of some phenomenon. Quite the contrary, the hermeneuticist recognizes his or her complicity in shaping the phenomenon, simultaneously affecting and affected by both the particular and the general, thus wholly embedded in the situation. In other words, the "object" of the hermeneutic inquiry is a moving target. As we study our conception of mathematics teaching, for example, our understanding of teaching—that is, the very "object" of our inquiry—changes.

And so, hermeneutic inquiry cannot be conceived of as a linear process. While we as-yet lack a word to describe the sort of path that might be taken through the research, terms such as recursive, circular, and reflective provide some sense of the process. Hermeneutics, then, seeks to undo our habit of "writing backward"—that is, of weaving narrative strands that serve to impose structure on and that enable us to extract meaning from an amorphous mass of, at the time of living through it, largely unformulated experience.

It is easy to see why hermeneutics is often perceived of as "philosophical" (in the derogatory sense). This description, while on one level appropriate (its purpose is to assault "common sense"), is unfortunate. As Smith states, "we never think or interpret 'in general' as a rhetorical activity that bears no necessary connection to the world at large."[34] We cannot extract or abstract our thinking (or our selves) out of the world we are thinking about. In consequence, a "clear split between subjective thinking and objective thinking is ridiculous because my subjectivity gets its bearing from the very world I take as my object"[35] and with which, to re-call an enactivist notion, I co-emerge.

In this writing, I have attempted to re-present this subject to object interplay and the general to particular movement in the structure on the document. Each chapter presents a cycle from the broad (the cultural) to the specific (the classroom); in each I have endeavored to enact the dialogical and circular nature of my and my co-researchers current understandings.

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[32] Quoted by Smith, ibid., 190.

[33] The reference to the "social sciences" rather than the "hard sciences" is deliberate. Reporting on a paradoxical shift in the two broad areas, Margaret Donaldson describes how, earlier in this century, social scientists endeavored to write themselves out of their research at the same time that physical scientists were compelled to write themselves in. In her words, "we see the strange state of affairs that, while prominent schools of psychology were trying to push consciousness out of their theorizing, physicists were finding they had to bring it in." — Margaret Donaldson, *Human Minds: An Exploration* (New York: Allen Lane The Penguin Press, 1993), 187.

[34] Smith, "Hermeneutic Inquiry: The Hermeneutic Imagination and the Pedagogic Text," 191.

[35] Ibid., 192.

I have also attempted to include a trace of the emergence of those understandings, but these traces are more often implicitly that blatantly stated. As such, at the outset, I provide an orienting statement as to the manner in which the guiding questions were developed and posed and the ways in which the research settings were "structured." Specific details of these elements are distributed throughout the text. The more general considerations follow.

The Hermeneutic Question

A theme running through this document is that, fundamentally, the nature of human experience is dialogical; my world is neither objectively fixed nor subjectively constituted, but negotiated with others with whom I find myself in communicative interaction. Truth, then, is not a static form which, after discovery or creation, takes on an autonomous existence; truth is always contingent, existing not in a single authority, but amid dynamic interaction and engagement. Truth is dialogical.

As such, the method used to investigate (*interpret*) a phenomenon needs to be similarly conversational—an idea which has implications not just for the style of the research, but for the questions that orient it.

Hans-Georg Gadamer[36] has provided us with a provocative exploration of this issue, arguing that the relationship between the research question and the phenomenon under study is not uni-directional, but reciprocal. Briefly, his suggestion is that the topic of investigation, at least in part, reveals the manner in which it should be investigated. Further, the phenomenon is shaped by the way we inquire into it—that is, by how we structure our question. Physicist Werner Heisenberg made essentially this same point in his famous statement, "What we observe is not nature itself but nature exposed to our method of questioning."

By calling to question our mode of questioning in this scientific age, hermeneuticists like Gadamer have often been accused of being anti-methodology. The accusation is unjustified. The hermeneuticist critique of conventional scientized approaches to research is *not* that those methods are wrong or inappropriate, but that they are narrow and inevitably lead to a particular (i.e., abstracted and ostensibly objective) sort of truth; they do not allow the researcher to move outside of a particular interpretive frame. Such research, we might say, is guided by the reductive question, "What is . . . ?" rather than the expansive, "What might be . . . ?" In contrast, successful hermeneutic research does not seek to close the doors of inquiry by arriving at some "answer" or uncovering some Truth. Rather, it seeks to open the doors wider, permitting both writer and reader to see their positions in a more open way.

In his analysis of the nature of the hermeneutic question, Gadamer develops the notion of "prejudice," by which he draws attention to the fact that what we hear and see is what we have pre-disposed ourselves to hear and see. With regard to research, then, what we come to understand is very much determined by the manner in which our orienting question is posed. Of course, in articulating the notion of prejudice, Gadamer is not recommending that we *avoid* pre-judgments, but that we seek them out and interrogate them. Since perceptions are enabled (just as they are limited) by such prejudices, the goal is not to negate but to transform—that is, to perceive differently.

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[36] Gadamer, *Truth and Method*.

This movement toward perceiving differently is only possible (and only necessary) when the orienting question is permitted to be similarly negotiated or interrogated. In the hermeneutic investigation, therefore, the questions are never fixed. Indeed, the hermeneutic question might better be thought of as an issue or a topic of wonder. It is an entry point for excavation, not an arrow for answer seeking. It presents an opportunity to unearth the heretofore hidden "truths" of how we arrived at our current place. The hermeneutic question is an idea, for as Gadamer explains, "every sudden idea has the structure of a question."[37] Moreover,

> the sudden occurrence of the question is already a breach in the smooth front of popular opinion. Hence we say that a question "occurs" to us, that it "arises" or "presents itself" more than we raise or present it.[38]

Such is the question that oriented this research: the issue of teaching as listening. I hope to illustrate in the following pages the sort of exploration and play that this notion invited. I also hope to give a deep sense of how, in the process of investigation, the orienting issue itself evolved through the dedicated, and ofttimes uncomfortable, interactions of the research participants (included among whom were fellow students, practicing teachers, professors, and unsuspecting acquaintances).

Such discomfort was to be expected because, as mentioned earlier, a hermeneutic investigation makes particular transformative demands on those involved. In seeking to interpret and to re-interpret experience, we are also seeking to affect how we stand in the world. A hermeneutic study, then, is fundamentally a moral undertaking. It makes no claims to the scientistic ideals of neutrality, objectivity, and generalizability. Rather, its goals are realized in deepened, practical, non-dogmatic, and consensual understandings among its participants: viable understandings rather than verifiable facts.

The Hermeneutic Conversation

"Hermeneutics" refers neither to a particular (instrumental) approach to research nor to a unified field of inquiry. Rather, it addresses a broad range of topics and issues. The unifying theme in hermeneutics is a persistent questioning of our taken-for-granted modes of speaking and acting.

It is thus that hermeneutics falls outside the conventional bounds of research methods—where the term "method" is understood to refer to a more-or-less static and rigorous procedure for attaining a pre-specified end. It is interesting to note that, according to the *Oxford English Dictionary*, this definition of "method" arose in the seventeenth century (that is, in the same era as Descartes, Galileo, and Bacon). Prior to that time, a method was "a shared (μετα-) way (-οδοσ)." That is, a method was an approach to knowledge that foregrounded the place of common action and accord rather than the questing to erect an autonomous truth—the process rather than the goal. Hermeneutics is a method in this pre-modern sense.

It is thus that the conversation—that is, reciprocal engagement in a topic of mutual concern—is generally identified as the site in which this sort of inquiry happens, for it is in dialogue with one another that our conflicting prejudices are uncovered and transformed. Subjectivity dissolves into participation in dialogue, the point of which, as

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[37] Ibid., 366.
[38] Ibid.

Crusius explains, "is not to hold a position against all challengers, but to listen, to allow one's opinions to be matured by opening oneself to partners in the dialogue whose horizons differ from our own."[39] Gadamer describes this process as a "fusion of horizons," where one's horizon is the ever-changing, historically- and situationally-shaped starting place of our thoughts and actions. In the conversation, Gadamer suggests, there is potential for such fusion as participants come to new understandings which are, at that moment of interactive unity, commonly held. A fusion of horizons is "an event of truth, a revealing-concealing that goes beyond the spontaneous, unscrutinized projections of preunderstanding."[40]

It is important to draw an initial distinction between a conversation and a discussion at this point. (The distinction is elaborated upon in the next section.) In the conversation, all of the participants are oriented toward deepening their understanding of the issue at hand. In a sense, then, the subject matter conducts the participants and there is a quality of self-forgetfulness as all concerned come to understand that they share in the *truth* of the interaction. The goal of the discussion, in contrast, is more toward the articulation of preformulated ideas, and so the subjects endeavor to exert some measure of control over the subject matter. The emphasis in the discussion is placed on the subjects' conceptual differences rather than on achieving a consensus. Rather than a forgetting of selves, there is a concretizing of subjective positions; horizons are not placed at hazard in the discussion.

Quite unlike the discussion, then, the conversation is fluid, meandering its way toward a destination that is not specific, but that will be commonly known. That the destination is unspecified and unanticipated is the strength of the conversation for, by being unconcerned with reaching a particular point (i.e., relinquishing the modernist desire for control)—by allowing the path to be laid down in walking— the participants are able to listen to the particularities that shape that path. The goal of the participants in a discussion, much in contrast, is often to remain rigidly in place, to not "be swayed."

A distinction might also be drawn between the conversation and the interview as a foundation of research. The latter, which literally means "between views," might be regarded as an even more radical version of the discussion, where the agenda might be rigidly pre-set (through the selection of questions, settings, etc.) and, very often, where the interpretive framework is laid out well in advance of any sort of interaction. There is little or no intention of having one's own views affected—and, with the usually impersonal and undemanding protocols—there is little danger of this occurring. The conversation is quite the opposite, both in spirit and in consequence.

There are, of course, dangers in suggesting that a hermeneutic investigation relies so fundamentally on the conversation. First, for example, in our modern setting, there is always a temptation to methodologize—a proclivity that can only serve to prevent rather than provoke conversation. Second, in a related vein, while one might avoid such attempts to induce conversation, there often remains a compulsion to explain how one comes to be aware that a conversation is taking place. In my own research, this particular compulsion has proven very troublesome, and has led to the dissolution of potentially fruitful interactions rather than to their promotion. This consequence should not be surprising, for the idea that one can be aware that one is in a conversation is in some ways self-contradictory; it presumes an awareness of one's self and one's subjectivity. It is

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[39] Crusius, *A Teacher's Introduction to Philosophical Hermeneutics*, 37-38.
[40] Ibid., 37.

precisely this detached, observer-like awareness that must be set aside in order to allow a conversation in the first place.[41]

Put differently, we can never be aware that a conversation *is taking place*. We can, however, be aware that one *has taken place*. When understandings have changed, when a new common sense has been established—when self and other have been altered—it has happened.

A third difficulty with the apparent reliance on the conversation is that the emphasis might suggest a disregard for other forms of research. A friend's reaction to my explication of hermeneutic inquiry illustrates this point: "So, you just have to go around talking to people and trying to come to some kind of common understanding."

Quite the opposite, hermeneutic inquiry relies on the planned as well as the unplanned, the expected as well as the fortuitous. Essential features of my own research included etymological searches of key terms, observation of mathematics classes, planning units and lessons, discussions of theoretical and practical issues in teaching, participation in academic conferences, broad and deep readings in a range of disciplines—not to mention extensive writing and re-writing. In particular, the exploration of historical accounts of various topics was of critical importance. Lacking a broad familiarity with the events and the perspectives that have shaped current practices and awarenesses, one's attempts to explore alternatives would probably be seriously constrained.

Further, day-to-day experiences, such as the one that I used to open this section, had "fuzzy" influences on the course of my life and, hence, and on the shape of this project. The extent and precise natures of these influences could never be determined. Nor would we want to do so in a hermeneutic inquiry. The goal is not to give a blow-by-blow account of how an understanding was reached, but to investigate how one's understandings might affect how one stands in the world.

The point here is not that this project was multi-faceted; all research projects are. Nor is it that some aspects were orchestrated while others were improvised; the same is true of the most rigorously controlled scientific experiment. The point is that, in order for the conversations to occur at all, there had to be considerable advance preparation and learning. In other words, I had a responsibility to ensure that I was capable of engaging in conversation—a responsibility which demanded not just that I had an interest in the topic, but that I have an adequate conceptual background to think of things differently. There was little potential for engaging others without the possibility of challenging their thinking. And there was little likelihood that I would be able to listen for the "watershed moments" that constantly, but quietly, presented themselves. In contrast to those more rigidly controlled modes of research, the goals of which are replicability, generalizability, and verifiability, the hermeneutic investigation seeks to understand the rich textuality of the unique amid the immediately present. The goal is thus to embrace happenstance rather than to "explain it away."

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41 This point may require some elaboration. I am not attempting to argue here that we, in some way, forget who we are or what we are doing when we converse with one antoher. The point, rather, is that the focal point of the conversation is neither you nor me, but a topic of shared interest. If the focus shifts to subjective or self-ish concerns, the mode of interaction tends to become something other than a conversation.

From the Visual to the Auditory (Revisited)

The movement away from culturally-privileged research methodologies toward a hermeneutic inquiry might be described as a shift from *looking* to *listening*.

We tend to use visual metaphors in our descriptions of scientific and scientized approaches. (A *glance* in a thesaurus for synonyms of such terms as "understanding" and "investigation" is most *revealing*.) Many science texts use the image of a disembodied eye to represent the scientific attitude, reinforcing the notion that, through vision, the subjective observer is separated from objective reality. The necessity of such separation is hardly surprising since the word "science" is derived from a term that meant "to cut apart." This scientific gaze, insofar as it is applied to issues in a social context, also tends to "freeze" phenomena. (This tendency is powerfully revealed in the pervasive use of *statistics*—derived from the Greek *states*, one that stops or steadies—as a basis of interpretation.)

In contrast, the hermeneutic text endeavors to avoid these distancing and fixing activities, and this is why explications of hermeneutics tend to employ auditory metaphors. The notion that *conversation* is foundational to the hermeneutic inquiry, for example, has a figurative as well as a literal relevance as we seek to locate ourselves in the wider conversations of history and context. Further, the goals of hermeneutic research are toward *attunement* and *harmony* amid the *noise* of existence. One is concerned with *theme*, with *tone*, with *rhythm*, with *resonance*. As such, one is constantly reminded that understanding, like sound, is fleeting and unfixable.

Further, unlike the isolating tendencies of vision, sound incorporates. Sound pours into the listener, whereas the object of sight exists outside the observer. Walter Ong (1982) elaborates on this point:

> Vision comes to a human being from one direction at a time: to look at a room or a landscape, I must move my eyes around from one part to another. When I hear, however, I gather sound simultaneously from every direction at once. . . . You can immerse yourself in hearing, in sound. There is no way to immerse yourself similarly in sight.
>
> By contrast with vision, the dissecting sense, sound is thus a unifying sense. A typical visual ideal is clarity and distinctness, a taking apart. . . . The auditory ideal, by contrast, is harmony, a putting together.
>
> . . . Knowledge is ultimately not a fractioning but a unifying phenomenon, a striving for harmony.[42]

It is thus hardly accidental that, in my initial explorations of how mathematics teaching might be re-cast in terms of listening, I felt a particular resonance with the *sound* foundations of hermeneutics. There is a certain harmony implicit in the statement that this project—this quest into teacher's listening—is hermeneutic, for the hermeneutic attitude is a listening attitude.

But, what is the nature of listening? Or, more fundamentally, how might we investigate the phenomenon of listening?

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42 Walter Ong, *Orality and Literacy: The Technologizing of the Word* (New York: Methuen, 1982), 72.

(Hermeneutic) Phenomenology

Hermeneutics reveals that our understandings, our perceptions, our actions, our experiences are objects that are always and already interpreted; they are constituted within our common language, they are enacted in certain settings, they are framed by particular webs of relationships. By questioning the terms, the traditions, and the texts that shape our understandings, hermeneutics raises the hope that we might begin to think differently about our selves and our situations.

Hermeneutics, then, holds as its focus the re-interpretation of already interpreted phenomena. But what of those phenomena that precede and invite interpretation—such as that which enables (or comes to be referred to as) our *listening*?

These sorts of questions demand a descriptive rather than a strictly interpretive methodology, and this is the realm of phenomenology—the "study of essences"—which concentrates its efforts "upon re-achieving a direct and primitive contact with the world."[43] Phenomenology investigates the nature of things and events, seeking to undercut our theoretically-sedimented and linguistically-constituted pre-conceptions; phenomenology "demands of us a re-learning to look at the world as we meet it in immediate experience."[44] The purpose is not to explain or control, but to bring us in closer contact with the world.

Historically, scientists have attempted to adopt positions of disembodied observers or disworlded minds that are parachuted into uncharted objective reality. Critiquing such positionings, Merleau-Ponty wrote, "The world is inseparable from the subject, but from a subject which is nothing but a project of the world, and the subject is inseparable from the world, but from a world which the subject itself projects."[45] We are complicit in the world, and so we can never really speak of it as removed from ourselves. Phenomenology, then, "does not produce empirical or theoretical observations or accounts. Instead, it offers accounts of experienced space, time, body, and human relations as we live them."[46]

Of course, such description must occur within language. Phenomenological accounts are therefore interpretive (hermeneutic) accounts as well. In attempting to get at the experiences that precede language, phenomenology inevitably faces the paradox of using language to push language aside. The text of the phenomenological account, then, is not used primarily to offer descriptions but to *point* at the experiences or phenomena that underlie those descriptions. Invariably, such an investigation involves extensive writing and re-writing as one endeavors to pull away from what has long been taken-for-granted in our hurried and unreflective ways of living. Such undertakings demand attitudes of patient attentiveness and persistent questioning, never allowing one's conceptions to be fixed or settled. An inevitable consequence of a phenomenological investigation is thus a renewed appreciation of the complexity that permeates the simplest, most straightforward experience.

Because phenomenology is the study of the intersections of human experience in consciousness, it can only be understood by participating in the phenomenological method. The next section, on the nature of listening, represents my attempt to do just that.

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[43] Merleau-Ponty, *Phenomenology of Perception*, vii.

[44] Max van Manen, *Researching Lived Experience* (Toronto, ON: The Althouse Press, 1990), 184.

[45] Merleau-Ponty, *Phenomenology of Perception*, 430.

[46] van Manen, *Researching Lived Experience*, 184.

Section C
Listening

> Our listening needs to learn receptiveness, responsiveness, and care.
> Our listening needs to return to the intertwining of self and other, subject and object;
> for it is there that the roots of its communicativeness take hold and thrive.
> — David Michael Levin[47]

It is interesting to note how often the term "listening" arises in the current social, political, and economic contexts. Over the course of a typical news broadcast, it is not unusual to hear political candidates promising to listen to their prospective constituents or warring factions demanding that their opponents listen to their claims. In a school staffroom, teachers comment that they are no longer listened to: pupils seem to have lost the capacity; parents and the government seem to have lost the interest. In the hallways of the same school, students offer a similar lament: no one seems to be listening to them.

In each of these cases, one of the concerned parties feels distanced from the other and each believes that this distance might be reduced by listening. Unfortunately, demands for such listening are usually made of others; the concern is with someone else's inattendance. It seems that we are wont to regard ourselves as capable listeners, and others simply as lacking this capacity. Consider, for example, how often one hears exchanges of the following form in the context of an argument or a heated discussion:

"If you would only listen!"
"No, you're the one who needs to listen!"

It is at first tempting to suggest that such exchanges indicate incompatible understandings of listening. In this chapter, I wish to argue against that idea: such understandings are not *incompatible*, they are *incomplete*, for they are founded on inaction rather than enaction. I approach the topic by developing a phenomenological account of listening, focusing not so much on what listening *is* but on what it might mean to think of ourselves as beings with the capacity to listen.

Underlying this exploration is the belief that the modernist notions which are implicit in most conventional conceptions of interpersonal communication are, simply put, false. We tend to take for granted, for instance, that we are insulated and autonomous individuals, that a mysterious substance called "information" can flow between us as we interact, that we are somehow in control of what is said and what is heard. In some current educational discourses, these ideas have coalesced into such notions as *voice* and *empowerment*—notions which acknowledge the ineffectiveness of communication in today's settings, but which reify instead of dismantle the modernist separations that underlie this ineffectiveness. Rather than seeking to promote conversation, we are compelled to perpetuate the model of competing monologues. The goal of such emancipatory discourse, it appears, is to promote listening—but by force. The reasoning seems to be that by developing more powerful voices, we will be able to reach across separations and we will be able to compel others to see things from our perspective.

In brief, we want people to think the way we think.

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[47] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 223.

In essence, this project is directed at proposing an alternative to this ideal. I contend that we do not need to amplify our voices in an effort to overcome chasms and walls. Rather, we need to realize that those barriers are not really there, and a deeper understanding of listening will enable us to dispel these pervasive illusions.

Our Collective Loss of Hearing

During the 1970s, a colleague and friend spent several years teaching in a remote village in Canada's far North. Early in her stay, several of the local Inuit women arrived at her door for an unannounced after-school visit.

Laura welcomed them into her home and, in true Western fashion, attempted to be a good host by serving tea and making light, but pleasant, conversation. To her frustration and concern, however, the collective response was one of prolonged and almost total silence. They sat in her living room, each quietly knitting or sewing. Not surprisingly, it was with the greatest relief that Laura bade them a good evening some time later.

Months after, when relationships were better established and Laura felt herself more a part of the community, she asked some of her new friends about that first encounter—in the process inadvertently revealing her belief that her visitors had been suspicious of her intentions and had thus dropped by to place her under scrutiny. Laughing at this suggestion, the women assured her that their actions had nothing to do with any sort of initial apprehension. Rather, they explained, they had visited to get to know her better. Further, they had not thought of their collective action as being intrusive. Holding different conceptions what it means to live together, they did not go to Laura's home to "visit" (in the Western sense of "dropping in" and departing without leaving a trace), but to dwell.[48]

My own interpretation of this story is that these Inuit held a certain disdain for the Western uses of discussion and surveillance as a means of introduction (if they were even aware of them). Far from helping us to *get to know* one another, in such necessarily shallow interactions we mask ourselves with convention. The resulting "conversations" are more concealing that revealing, serving to underscore our respective subjectivities rather than helping to dissolve them in communicative action. It is by listening—by attending to the person's action and situation, and not just to her voice—that one comes to know the other.

It is toward a similar sense of listening that this section is directed, a listening that is neither limited in focus to the verbal nor itself held silent. And a listening that seems all but lost—over*look*ed—in our society.

For, as argued in the preceding section, ours is a culture that favors the visual over the auditory, and this characteristic of modernist societies represents a dramatic departure from earlier traditions. Walter Ong,[49] for example, in his account of the recent movement away from spoken (oral) traditions toward written (visual) ones—a shift that accompanied the "scientific revolution"—suggests that not only has there been a loss of

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[48] I am attempting to invoke Heidegger's sense of "dwelling" here—a formulation which is more toward a dynamic and affecting participation with one's *ecos* (home; living place). To *dwell in* is not simply to *live among*. The latter evokes senses of separate existences (ones which become associated only because of their proximity), whereas the former implicates all present in mutually-specifying and co-emergent processes. See Martin Heidegger, *Basic Writings* (San Francisco, CA: Harper Collins, 1977).

[49] Ong, *Orality and Literacy: The Technologizing of the Word*.

status for the auditory, there has emerged a contempt for the spoken, the heard, and the listened to.

Belenky et al., in a similar vein, point to the "allocation of listening to women,"[50] contending that our privileging of looking over hearing is tied inextricably to our modernist favoring of the masculine over the feminine. Consequently, within our culture, there is a pervasive use of visual metaphors to describe the many *facets* of education. We *see* learning as gaining *insight*, intelligence as *brightness*, investigation as *looking*, understanding as *seeing*, opinions as *perspectives* or *views*, hopes as *visions* and (very often) teaching as super*vision*. More broadly, tendencies to associate truth with *light*, believing with *seeing*, and objectivity with the distance afforded only to the observer, point to the overwhelming domination of vision over the other senses. Contrasting visual with auditory metaphors, Belenky et al. write:

> Visual metaphors encourage standing at a distance to get a proper view, removing—it is believed—subject and object from a sphere of possible intercourse. Unlike the eye, the ear operates by registering nearby subtle change. Unlike the eye, the ear requires closeness between subject and object. Unlike seeing, speaking and listening suggest dialogue and interaction.[51]

We tend to stand back in order to see and to move nearer in order to hear. Correspondingly, there is an element of discomfort associated with being watched, but we generally want to be listened to—in part, at least, because of the interaction afforded by listening. Whether I am the "listener" or the "listened to," I participate in a very different way than when I am the "watcher" or the "watched." In particular, because we are unable to shut off our hearing with the ease that we can close off our seeing, attempts to *not* hear often result in being compelled to listen more attentively. As suggested by the imperative, "Close your eyes and listen," we are inevitably immersed in the sonorous field of the situation.

This is the idea that Laura had missed during the first visits of her Inuit neighbors. They had not come to hold her at a distance in their gaze, but to draw her into their circle in their listening.

Looking for listening

Since beginning this study of listening, a favorite activity has been to "eavesdrop" on various sorts of interactions. These covert activities have not been limited to classrooms; settings have included offices, restaurants, airplanes, and conference halls. What has come as a surprise, and what may be more an indication of my own inability to escape the privileging of vision, is how easy it is to *see* listening.

There is a particular bodily aspect to listening, a visible orienting to the subject of the discussion. When two persons converse, for example, it can be *seen* that they are listening to one another as the actions of their bodies become bodily *inter*actions. They lean into and reach out for one another, momentarily unaware that they are violating the Western taboos on proximity, touch, and extended eye contact. They seem to focus in a way that suggests they are oblivious to the noise around them; they attend to each word

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[50] Mary F. Belenky, Blythe M. Clinchy, Nancy R. Goldberger, and Jill M. Tarule, *Women's Ways of Knowing: The Development of Self, Voice, and Mind* (New York: Basic Books, 1986), 167.
[51] Ibid., 18.

and to each action as though nothing of importance occurred prior to the discussion and nothing of importance awaits them at its end. They are unconcerned that their voices are perhaps too loud, that their bodies are too animated.

Listening, then, need be neither motionless nor silent (although more often than not, it seems, it is precisely this sort of inactive attention that is demanded of students by teachers). Of course, the listener may assume this posture, but it is something other than an audience's lack of motion or their silence that makes us aware that they are listening. In the classroom, for example, as the novel is read or the mathematical principle emerges, the teacher knows the students are listening not because they have ceased to move but because a certain rhythm or harmony is established—there is an awareness that each is immersed in and conducted by the same subject matter. The gazes are fixed not on the teacher nor on one another, but on that which is among them.

Important qualities of listening, then, are that it be active and participatory, and an immediate implication is that the listener cannot be held silent. (She may choose not to speak, however.) As one listens, one questions, one challenges, one smiles, one frowns. We often characterize such interactive action as a "forgetting of self"—an intriguing notion, but one that I believe misdirects our attention. Listening more involves a dissolution of static notions of the self, permitting a re-membering of intersubjective awarenesses—a "joining of minds."

The Conversation

In my listening to listening, I have also begun to notice a clear distinction between two sorts of interactions which have a similar appearance but a very different texture: the discussion and the conversation. The distinction between these communicative forms is not so much evident in the words spoken or in the topics addressed as it is in the manner in which the participants listen to one another.

The discussion might be characterized as "coordinated action"[52] in which the respective speakers are attempting to impose their perspectives on the other. Their concerns, then, are for the articulation, explication, and defense of their own views. In more adversarial forms of discussion—those more resembling debates—more energy is directed toward attending to the other's perspectives, but this attendance is with the goal of dismantling those opinions, not for understanding their origins, let alone for seeking consensus.

The conversation, in contrast, is less oriented to pointing out difference and more concerned with arriving at shared understandings. Put otherwise, the discussion is an analytical rhetorical structure; the conversation is a place of listening. An analysis of the origins of the two words helps to make this point clearer: To *discuss* originally meant to "shake apart," and its emphasis on separation continues to echo in our current use of the term. To *converse*, much in contrast, had a meaning more toward "to live with" or "to keep company with." This is why we use "conversation" when referring to interactions with friends and "discussion" when speaking of meeting with strangers, professional colleagues, and business contacts. The conversation-discussion distinction also points to a, perhaps unfortunate, trend in teacher-student interactions: as recent developments have alerted us to the importance of active interaction while learning, we have tended to opt for

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[52] Here I am borrowing from Charles Taylor's "The Dialogical Self," in *The Interpretive Turn: Philosophy, Science, Culture*, eds. David Hiley, James Bohman, and Richard Shusterman (Ithaca, NY: Cornell University Press, 1991).

"classroom discussions" rather than conversations. Not surprisingly, there doesn't seem to be much listening happening within such settings, although (in my own experience with classroom discussions, at least) the demands that others listen tend to increase dramatically.

The conversation, thus, offers a rich territory for an exploration of listening. As Taylor explains,

> [Conversations] move beyond mere coordination and have a common rhythm. The interlocutor not only listens but participates with head nodding and "unh-hunh" and the like, and at a certain point the "semantic turn" passes over to the other by a common movement. The appropriate moment is felt by both partners together in virtue of the common rhythm.[53]

Taylor contrasts such "dialogical acts" with "monological acts"—acts of a single, ostensibly autonomous and isolated, agent. Taylor's use of "dialogical" is, I believe, similar to Varela et al.'s use of "co-emergent," suggesting that a conversation is more than an intertwining of two separate voices (or, in Taylor's terms, two "monologues"). Rather, the conversation involves a merging of subjectivities as, together, we are conducted toward new questions and new understandings. Unlike the discussion, which lacks the qualities of rhythm and of "living with the other," the conversation's path is neither predictable nor controllable. Nor would we want to prescribe its route or its outcome because, again in contrast to the discussion, the "purpose" of the conversation is as much the act of conversing (i.e., "living with others") as it is the development of a deeper understanding. There is no winner, no gaining of the upper hand, no final word, no compulsion to stick with the topic. Rather, the conversation allows us to move freely and interactively toward those questions that animate us while enabling us to explore not just the topics that emerge, but why such topics capture our interest in the first place.

Gadamer has written insightfully on the nature of the conversational relation. In his formulation, the conversation is a triad involving you, me, and the topic or subject matter. The subject matter exists only in the conversation[54]—neither in you nor in me, but between or about us—and we are "conducted" by it.

For Gadamer, the conversational relationship is an intimate one—an idea that is echoed by Merleau-Ponty who suggests that human interaction involves a merging or an intercorporeality.

> [As I listen to another, my body] discovers in that other body a miraculous prolongation of my intentions, a familiar way of dealing with the world. Henceforth, as the parts of my body together comprise a system, so my body and the other person's are one whole, two sides of one and the same phenomenon, and

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[53] Ibid., 310.

[54] This use of the term "subject matter" may seem somewhat ambiguous in the context of a piece of writing which is developed around a particular "subject area."Here the term is not intended to refer to the "subject matter of mathematics" or to any other discipline (although such a reference might be an interesting metaphorical extension of the current idea). Rather, the point being made is that the conversation is more than the sum of its parts; it has an integrity that exists in the joint action of its participants—an integrity that gives rise to actions and insights which would not otherwise have come about. To say that "the subject matter exists only in the conversation," then, is not intended to suggest that the subject matter lacks a history or that is dissipates when the conversation ends. It is to say, rather, that we are capable of engaging in joint action, and that action is itself the subject matter of the conversation.

> the anonymous existence of which my body is the ever-renewed trace henceforth inhabits both bodies simultaneously.[55]

We are thus *joined* in conversation, a theme that is common in the writings of both Merleau-Ponty and Gadamer. Elsewhere, for example, Merleau-Ponty describes the communicative act as "one system with two terms (my behavior and the other's behavior) which function as a whole."[56] Husserl, similarly, described such coordinated action as a phenomenon of "coupling" where, according to Merleau-Ponty, the notion of "coupling" is "anything but a metaphor."[57] He goes on to explain that it is our capacity to perceive—to sense and to make sense of—the other that enables this phenomenon: "In perceiving the other, my body and his are coupled, resulting in a sort of action which pairs them (*action à deux*)."[58]

Maturana and Varela,[59] working from a basis in the biological sciences, have arrived at a similar formulation. In their terms, the interactive unity of the conversation involves a "structural coupling" that brings about a system of higher order. The conversation is thus not a "third thing" that is made up of two people; nor is it something happening between them. Such interpretations—like the suggestions that the human body is just a compilation of organs which, in turn, are mere assemblages of cells—miss the essential element that bodies and organs (and conversations) have integrities proper to themselves that are analogous to the integrities of their subsystems.[60]

Complementing Gadamer and Merleau Ponty, then, Maturana and Varela would agree that, in conversation, we set aside our illusions of subjectivity, allowing a collective consciousness to emerge. In this relational unity, we become capable of greater insight and deeper understanding,[61] capable even of cutting beneath the conscious intent of the speaker.

A goals of the conversation are to deepen understanding and, in that deepening, to create knowledge. It "has a hermeneutic thrust: it is oriented to sense-making and interpreting that notion that drives or stimulates the conversation."[62] The key to such sense-making, that which enables the interpretation, is listening, itself hermeneutic. It is

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[55] Merleau-Ponty, *Phenomenology of Perception*, 354.

[56] Maurice Merleau-Ponty, *The Primacy of Perception* (Evanston, IL: Northwestern University Press, 1964), 118.

[57] Ibid.

[58] Ibid.

[59] Maturana and Varela, *The Tree of Knowledge*.

[60] Further to this point, Maturana and Varela also argue that our language is the analog of the physical substances that pass between other same-species organisms in a process is referred to as *trophallaxis*. Trophallactic substances are constituent parts of the organisms and make possible their structural coupling.

I believe this analogy may assist us in developing a deeper sense of the conversational relation. Consider, for example the implications of the following sentence (from Gadamer, in *Truth and Method*) if "trophallaxis" is substituted for "language": "Every conversation presupposes a common language, or better: it creates a common language" (p. 378). The implications for our capacities to hear and to listen are profound.

[61] As will be elaborated in Chapter 4, *understanding* is understood herein in terms of our capacity for action. A "deepened understanding" is thus, in Maturana's and Varela's terms, an expanded "sphere of behavioral possibilities." In colloquial terms, the point being made with regard to the conversation is that "two heads are better than one"—a notion that the enactivist might rephrase as, "two heads, in structural unity, have a far greater range of potential action than the two heads acting separately."

[62] van Manen, *Researching Lived Experience*, 8.

our capacity to listen—that is, our ability to attend to and to interpret what is said—that makes conversation possible. Levin suggests:

> When listening really echoes and resonates, when it allows the communication to reverberate between the communicants, and to constitute, there, a space free of pressure and constraint, *it actively contributes, quite apart from the speaking, to the intersubjective constellation of new meaning*, meaning actually born within this intercorporeality; and it promises, because of this, the achievement of mutual understanding—if not also consensus.[63]

The conversation, enabled by our capacities to listen, is a "meeting of (embodied) minds."

Listening is not a technique

Robert Pirsig describes the title character of his book, *Lila: An Inquiry Into Morals*, in the following way:

> What he'd told her . . . was valuable if she'd been listening. But she wasn't. She wasn't a listener. She had a fixed set of static patterns of value and if you argued with her, she'd get mad at you.[64]

The statement, "She wasn't a listener," is immediately comprehensible. In so describing Lila, Pirsig is suggesting not that she was incapable of hearing, but that she was not normally open to others and to their ideas, limited by her "fixed static patterns." Listening thus is not primarily an act, it is an orientation. Everyone is capable of listening, but few, it seems, are in the world in a listening way. Listening, then, is a way of being in the world, an "*ontologically oriented* capacity"[65] that is directed toward bringing ourselves and others into being.

We are all acquainted with people who are not listeners. When we talk with them, they might ask the correct sorts of questions and perhaps even display the appropriate mannerisms, but our contributions either seem to be ignored or misinterpreted. We get an uneasy feeling because, even though we find ourselves within the interaction, it does not seem that we are part of it. We feel rushed, unheard, not listened to, excluded—not present. We quickly become unwilling to "share" even the most mundane thoughts.

Nevertheless, in keeping with the modernist tendency, efforts have been made to reduce listening to a technique. Text after text on the topic of improving listening skills is now available, almost all of them suggesting that success (usually in business) is tightly linked to mastering this ability.[66] These manuals, it seems, are founded on the premise

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[63] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 181.

[64] Robert Pirsig, *Lila: An Inquiry into Morals* (New York: Bantam Books, 1991), 158.

[65] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 17.

[66] In this university's library system, there are literally hundreds of resources on the topic of improving listening skills. Some titles include: *"I hear you": How to Use Listening Skills for Profit*; *Listening Made Easy: How to Improve Listening on the Job, at Home, and in the Community*; and *A Manager's Guide to Speaking and Listening*.

Interestingly, *most* of the resources on listening are housed in the education library—and almost all of those focus on students' listening skills (i.e., means of improving these skills or the correlation between listening abilities and academic achievement. Those few that deal with the teacher's listening focus on how one might listen to appraise and assess student articulations.)

that the acquisition of particular skills will change the person and the sorts of relationships that person is able to maintain, in contrast to the perspective that how one listens—and hence, the nature of one's relationships—emerge from the way one is already standing in the world.

One notable contribution was made by Carl Rogers[67] in the development of his "Active Listening" program—a system by which particular skills, intended to enable the listening of psychologists and counselors, were identified and described. Unfortunately, in many cases, Rogers' program was misinterpreted as focusing on the development of these skills rather than on the development of one's listening. A guidance counselor and colleague was among the victims of this technical reduction. Shortly after being appointed to the position of school counselor (by an administration that was impressed with the sorts of relationships she had with students), Elaine attended a series of seminars and inservices where she was to develop a better understanding of her role. Based on what she had learned in these sessions, she tried "active listening" by paraphrasing, repeating, requesting clarification, and affirming student articulations. The general response was one of confusion, frustration, and anger—and Elaine quickly recognized the impairing effect of focusing on the skills rather than on the listening. As is typical of modernist projects, this effort resulted in a separation and a distancing rather than a facilitation of communicative interaction.

And so listening cannot be reduced to a set of prescriptions or guidelines. It is something that we enter into, something that we are, emerging from our occupation with others and with their meanings.

Listening as *Embodied Action*

Listening is a way of being which surpasses our efforts to formalize or articulate. It is not so much a conscious effort as it is a way of participating with others.

Charles Taylor[68] writes of two sorts of action: formulated and unformulated. The former he describes as those thoughts, behaviors, and bits of knowledge which we have written into the text of our experience—those we are aware of, speak of, and tend to link in narrative or causal chains. Such formulated actions, he argues, represent only a small portion of our total action, even though they dominate our conscious awarenesses. The bulk of our moment-to-moment living is a matter of unformulated action—a negotiated movement through an interactive world during which our knowledge of that world and our way of being in that world are continuously enacted. The evidence of such knowledge and understandings is our survival, not our ability to identify or explain (narrate) our actions in formal terms.

Understood in the context of our daily lives, then, listening is enacted and unformulated. An important implication of this idea is that listening cannot be considered as merely an auditory capacity that can be understood in behavioral science terms. We listen not just with our ears and minds, we listen with our bodies. It is an activity of all the senses, attuned not only to the text of the conversation, but to the subtexts, the contexts, and the textures. It is not strictly—nor even primarily—an academic or intellectual activity; it is a fully human endeavor that also evokes physical and emotional

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[67] See Richard A. Leva, *Psychotherapy, The Listening Voice: Rogers and Erickson* (Muncie, IN: Accelerated Development, 1987).
[68] Taylor, "The Dialogical Self."

responses. As Merleau-Ponty suggests, "I echo the vibration of the sound with my whole sensory being."[69] We listen with our ears, with our eyes, with our touch, with our stomachs, with our bodies, bringing the collected weight of our experience to our emerging understandings.[70]

Listening is not the same as hearing

Part of my research has involved making audio-recordings of classrooms. Each time I sit to transcribe bits of the teacher-student interactions from one of these recorded lessons, I am struck by the muddle of sounds the machine has captured. There are rustling papers, falling pens, and textbook covers slapping against desktops. There are whispers, sighs, and laughter. But when I was there, I was unaware of this hum of the classroom; I quite simply did not hear these sounds.

I am able to induce the same phenomenon at this moment as I pause from my writing to listen, becoming once again aware of the sounds in which I am immersed. It seems that only when my attention is drawn or directed to particular sounds—like the rumbling of the traffic outside my window—that I am able to hear them.

Moreover, returning to the recordings, as I listen in on the classroom through the mechanical ear, I must struggle to hear particular voices that are woven unevenly and that are tangled with one another. Cut off bodily, I am unable to hear interactions that were easily heard when I stood in the classroom. Cut off bodily, I cannot enter the ebb and flow—to become part of the tone and the tempo—of the lesson. I can do little more than re-hear a now blurred rendering of that lesson. I can do no interrogation, and so I can do little listening.

Hearing and listening, then, are different phenomena. Consider, for example, the difference in intended meaning between the two statements: "I can't *hear*" and "I can't *listen*." In uttering the former, my concern is that the sound isn't loud enough. It is something I say when I want to hear but, for whatever reason, cannot. The concern is strictly sensory.

In uttering the latter, however, I am suggesting that *I am able* to hear the sound without difficulty. When I say, "I can't listen," I'm not saying that I *can't* hear but that I *won't* hear. To make this point clearer, the statement "I can't hear" is often followed with "Turn *up* the volume." But the statement "I can't listen" tends to be accompanied by requests more along the line of "Turn *down* the volume." Listening is thus a capacity which is founded upon hearing but which goes beyond hearing. It is orienting (we listen *to* something) and oriented (we listen *for* something). Hearing, in contrast, lacks such intentionality.

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[69] Merleau-Ponty, *Phenomenology of Perception*, 234.

[70] Further to this point, it is important to emphasize that by describing listening in this manner, I am not suggesting that it is a "meta-awareness"—a perceptual capacity that can somehow move outside of itself and assume a detached, objectifying stance. Quite the contrary, such "meta"-notions are antithetical to listening. To use the relatively popular idea of *metacognition* as an illustrative comparison, the belief that we can somehow stand above our own cognitive processes and, in so-doing, to enable them, has the effect of underscoring rather than erasing the dichotomous traditions in which much of educational discourse is mired. Alongside mind/body, thought/action, knowledge/knower, and self/other, we are urged to include thought/metathought.

A comparison to the visual—to seeing and looking—may be helpful here. *Sight* is the sensory capacity; looking is the intentional action through which particular "objects" are pulled into focus (brought forward). Hearing and seeing, the sensory capacities, present us only with an undifferentiated background. It is our capacity to draw something out of that background of experience, to focus on it, and to bring it into ourselves—that is, to listen and to look—that enables our perceptions.

The first distinction between hearing and listening is thus made. Hearing is the sensory capacity that underlies our ability to listen. In the classroom, what I heard was determined by what I was listening for: the teacher's questions, the students' answers, the range of pitch and tone in their voices. In one sense I heard everything, because I was aware of no gaps in my perception; the experience was seamless. But in truth, I heard hardly anything.

This also points to a second important distinction between hearing and listening, one that is suggested by the phrases, "I hear you" and "I'm listening." At a recent parents' meeting, a school principal responded to each of the parents' concerns with "I hear you." But is was clear to every parent that she was not listening. In repeating, "I hear you," she was suggesting that she understood all that was being said, that the speakers' meanings were apparent to her, that there was no need for further listening. Put differently, the parents were not participating in a conversation; they were *having input*.

Hearing presumes understanding, and when we cannot comprehend someone, "we can't hear a word he's saying." In contrast, the statement "I'm listening" implies a recognition of the preeminent role of interpretation in our interactions. It is when we perceive gaps in our understandings that we are compelled to listen. Indeed, when someone challenges or fails to grasp the point we are attempting to make, we do not respond with, "There is a problem with your hearing," but by saying, "You're not listening." The distinction is an important one because much of our interaction tends to be undertaken in "hearing mode" rather than "listening mode"—that is, on a knowledge of the other's subjectivity, but not on an awareness of the intersubjective bases of emerging conceptualizations; on knowing, but not on understanding. While perhaps not inappropriate for much of our daily existence, the consequences of operating strictly in the "hearing mode" within the classroom, as will be developed later, can be devastating.

Listening for—Our listening is oriented.

Listening implies an attunement to the "voices" of others in all their richness. But it would be a gross overstatement to suggest that, by listening, we can somehow transcend the constraints on our perceptions. While I believe it a route to greater openness and richer relationships, we must acknowledge that our listening always and inevitably occurs against the backdrop of personal histories that are set in and shaped by cultural, historical, social, and environmental factors. It is here that the notion of "voice" as a unified projection of the speaker begins to break down, because one's voice can never be singular. Rather, it seems, it is more appropriate to think of our selves as choruses of voices or—to use Gadamer's term—as *conversations*.

In other words, persons are never merely individuals, "they are always also representatives of institutional power, bringing with them a multiplicity of vested interests—and many virtually inaudible agendas."[71] Each of us carries not only the

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[71] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 111.

history of our personal experiences, but the accumulated experience of the culture in which we are embedded. (It is thus that, merely by acting, we are participating in the evolution not just of our selves but of our society.) These experiences simultaneously enable and impair our listening—at the same time facilitating interpretation and limiting the possibilities for that interpretation.

But the central point is that listening is not a matter of accurately representing auditory input. Quite the contrary, what we listen for and what we hear is primarily a matter of what we expect or anticipate.

Philosophers and scientists alike have elaborated upon this point. Gadamer, for example, talks about the "prejudices" we inevitably bring to our listening and observing. In his conception, these prejudices are not negative, but necessary; perception is not possible without some sort of anticipation or pre-judgment to impress on what is sensed.[72] And Varela et al.[73] discuss the implications of a provocative study of human perception. Briefly, subjects were presented with various stimuli while their brain functions and their retinal activity were monitored. To the researchers' surprise, there was little correspondence between the qualities of the stimulus and the activity in the brain. However, there was a very strong correlation between retinal activity and brain function, indicating that "the flow of information" was not primarily from the object to the mind (from the outside to the inside) but from the brain to the eye. In effect, this result demonstrates that one's perception is more the product of what is "projected" than what is actually "sensed"—that the observers' perceptions are determined by their structures.

This inference is supported by a growing collection of case histories of persons whose sight has been restored after long-term blindness. In an account of one such incident, Oliver Sacks[74] describes how Virgil, through a cataract operation, regained the vision he lost several decades earlier. What is surprising about this case is that the immediate consequence of overcoming his blindness was despair and depression rather than happiness, for, although Virgil was not longer blind, he could not yet see. When the bandages were removed from his eyes, he was bombarded with a frightening mélange of undifferentiated images which—lacking "prejudice" or interactive experience with this aspect of the world—he could not reduce or translate into meaningful sights. He was thus trapped in the visual equivalent of "noise."[75]

Of course, these studies focused on vision, but there is no reason to suggest that hearing and the other senses are differently constituted. In an auditory analog of Virgil's story, for example, a friend who recently began to wear a mechanical aid to compensate for a rather severe hearing loss found herself unable to "screen out" unwanted noise when in a restaurant. Compelled to hear everything, she could hear nothing, and soon chose to turn off the hearing aid. She has since learned how to listen with it. I experience a

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[72] An illustrative example of this phenomenon occurred when a teacher was introducing himself to a new group of students. One student, named "Yoofi," was asked to repeat his name several times, and then to spell it, before the teacher was able to hear it. Lacking appropriate pre-judgments (which could only be found in phonetic analysis), this teacher's listening was constrained.

[73] Varela et al., *The Embodied Mind*.

[74] Oliver Sacks, "A Neurologist's Notebook: To see and not see," *The New Yorker*, 10 May 1993, 59-73.

[75] This account might also be used to further explicate how our perceptions of the world are *enacted*. Vision, for example, is clearly not the biological equivalent of using a camera to capture images and to re-present them in another context. Rather, the capacity to see (like each of the other senses) emerges as we live through and interact with the world. Another interesting point is that it might be more appropriate to speak of the development of vision as a process of learning how *not to see*, because far more is disregarded than is noticed in our seeing.

related—but in many ways the opposite—phenomenon when I visit noisy restaurants. At particular moments I hear fragments of background music which, for the most part, go unnoticed beneath the layers of other sounds. Occasionally one of those fragments "registers" and I am able to identify the tune. From that instant, I can hear the whole song, with remarkable clarity, despite the noise that prevents others from hearing it. Put differently, once aware of what I should be hearing, I can hear it. When unaware, the song goes unheard.

A third illustrative example emerges from this study of listening. As I *listen for listening*, I hear the term everywhere—voiced in every politician's promises, embedded in every text. Like my name mentioned across a noise-filled room, the word "listening" demands that it be heard.

In brief, then, we are compulsive sense-makers and, discounting some sort of handicap, have become adept at pulling bits from a sea of sensorial possibilities. Hearing is really more a matter of *not* hearing—a truism that is evident in those frustrating interactions where you have attempted to make a point but, regardless of what you say, the other person hears the wrong thing. "He hears only what he wants to hear."

The point I am trying to get at, then, is that "perception" and our "perceptual capacities" are far more than means of "taking in the world." Quite apart from the capacity to "pull in sound," hearing is more a process of imposing order on noise—of bringing forth a sound. To listen, then, is to subject our hearing perceptions to scrutiny, endeavoring to disrupt the "taken for granted" which precedes, constrains, and (in effect) determines those perceptions. "Listening" does not suggest an ability to transcend such constraints, but a willingness to "unfix" our selves or to position our selves differently.

The phrase "listening for" points at the inevitability of approaching interactions with a particular set of expectations or biases. That we cannot help but take a particular stance in our listening is usually revealed very quickly when my background differs markedly from that of my partner in conversation. In the extreme case, when languages differ, listening is reduced to attempts to interpret simple signals because there are few shared signs. But conflicting interpretive frames present even more imposing barriers to listening as they bring about a reluctance to adopt the other's stance. Similarly, if the subject matter is not something that commands my interest, my listening becomes labored, more easily distracted, and sometimes resentful.

But what is listened *for* is not strictly determined by personal prejudices. What we are able to hear (understand) is also dependent upon the context which provides us with clues to interpret the speaker's words and actions. It is thus that the same statement made in a different setting can take on a new meaning—or perhaps lose meaning entirely. This particular phenomenon is well illustrated by referring to the effect of replaying audio-taped interactions. Like re-reading a book or re-viewing a landscape, such re-listening brings with it something new, in part because the context has changed, in part because the listener herself is different. And so, inappropriate interpretations are not always a consequence of one's inability to listen; they may emerge from a failure to attend to contextual clues.

Listening to—Our listening is orienting

As suggested earlier, listening is intentional. It is directed toward a particular "object"—that is, toward bringing it forth out of an undifferentiated background of experience. In listening, I am drawing an object into myself, the subject, and so we are

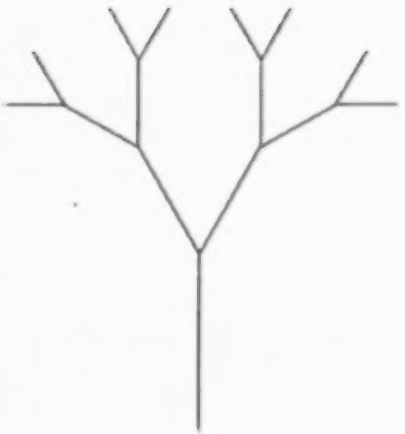
brought forth together; we are intertwined in our being and becoming; we co-emerge. Thus, an orientation to listening brings with it an awareness that there can be no rigid subject-object split. Additionally, this point illustrates that listening benefits the listener as well as the listened to.

But what is the nature of this "object" of listening in the mathematics classroom? At first, there is a temptation to suggest it must be either the speaker or the subject matter. Arguably, our proclivity to answer in these terms betrays persistent Cartesian perspectives on subjectivity and knowledge—perspectives that are challenged by Gadamer's, Merleau-Ponty's, and Maturana and Varela's accounts of the communicative relationship.

These are issues that I will deal with in later chapters as I explore the natures of mathematical knowledge and the teacher-students relationship. For the moment, I will point to an answer to the question, What is the object of our listening?, by suggesting that this object is analogous to a "game." As in other forms of interaction, details such as the participants, the setting, the rules of play all serve to proscribe boundaries on the game, but they are not the game. Rather, the game exists only in the playing. Similarly, the "object" of listening exists in the interplay of the participants, the setting, and the subject matter.

Chapter 2

AN EAR TO THE GROUND
The Subject Matter



Mathematics is not a book confined within a cover and bound between brazen clasps, whose contents need only patience to ransack; it is not a mine, whose treasures may take long to reduce into possession, but which fill only a limited number of veins and lodes; it is not a soil whose fertility can be exhausted by the yield of successive harvests; it is not a continent or an ocean whose area can be mapped out and its contour defined; it is as limitless as that space which it finds too narrow for its aspirations; its possibilities are as infinite as the worlds which are forever crowding in and multiplying upon the astronomer's gaze; it is as incapable of being restricted within assigned boundaries or being reduced to definition of permanent validity, as the consciousness of life.

—James Sylvester [1]

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[1] Quoted by John D. Barrow, *Pi in the Sky: Counting, Thinking, and Being* (Oxford, GB: Clarendon Press, 1992), 121.

Section A

Mathematics

> The world is out there, but descriptions of the world are not.
> — Richard Rorty[2]

The "nature of mathematical knowledge" has been a topic of debate within the field of mathematics education from its inception, but it is only recently that the issue has taken centre stage alongside discussions of learning processes and teaching approaches. This rise to prominence has been a welcome development in a field where research projects have often begun with definitions of mathematics that are given in terms of the contents of curriculum manuals.

In this section, I deal specifically with the question of mathematics; in subsequent sections the discussion is broadened to include issues of curriculum-making and preparation for teaching. Consistent with the investigative framework outlined in the last chapter, these discussions will be based on and will provide an elaboration of enactivist theories, this time focusing more specifically on ecological notions.

This task is not an easy one, for mathematics itself—or our privileging of it—has been cited as a primary contributor to our "ecological crisis,"[3] and this is an accusation that I am inclined to support. Nevertheless, I believe it is possible to describe an "ecological mathematics" that is aware of the relational world from which it emerges and that is sympathetic to the natural world onto which it is imposed. I approach the topic through a brief history of the discipline in which some of its major transitions are identified. The goals here are twofold: to trace the evolution of the modernist conception of mathematics—that of a static, formal, hierarchical, and truthful body of knowledge—and then, by attending to our own traditions and to recent developments within mathematics itself, to explore alternatives to this modern conception.

"Ecological"

I will begin by drawing a distinction between the terms *environmental* and *ecological*. Although not synonymous, the two words tend to be used interchangeably in reference to the plethora of problems faced by our modern society. Many commentators[4] contend that this confusion has contributed to the obfuscation of the causes underlying these problems. The crises, they argue, are not *environmental*, but *ecological*.

The term "environmental" is used to direct attention toward our *environs*—our surroundings—and hence away from ourselves. This tendency gives rise to an immediate problem. As Wendell Berry explains, "once we see our place, our part of the world, as *surrounding* us, we have already made a profound division between it and ourselves."[5]

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2 Rorty, *Contingency, Irony, and Solidarity*, 5.

3 See, for example, Murray Bookchin, *The Philosophy of Social Ecology: Essays on Dialectical Naturalism* (Montreal, PQ: Blackrose Books).

4 See, for example, Gregory Bateson, *Steps to an Ecology of Mind* (New York: Ballantine Books, 1972); Berry, *The Unsettling of America: Culture and Agriculture*; David W. Orr, *Ecological Literacy: Education and the Transition to a Postmodern World* (Albany, NY: SUNY Press, 1992).

5 Berry, *The Unsettling of America: Culture and Agriculture*, 22.

Describing a crisis as environmental, then, leads to attempts to develop (usually scientific) solutions—in effect, to fall back onto the same mode of thinking that precipitated the original difficulty. Implicit in the notion of "environmentalism" is a reification of the Cartesian separation of individual and world. These are two distinct categories, connected only by chains of causality which are generally perceived as unidirectional.

Identifying a problem as *ecological*, in contrast, places it in our οικοσ (*oikos*), our household—that is, in the web of relationships in which we find ourselves and against which our identities are established. The focus thus shifts from outward gazes to a reexamination of the assumptions and the actions that gave rise to the crisis. *Ecology* is about interrelationships and interconnections. It involves an attunement to codependencies, mutual affects, and co-determinations—in essence, to the fundamental intertwining of all things. When we speak of *ecology*, then, we speak of everything that shapes our being—their effects on us and ours on them. Folding back to earlier discussions of enactivism, hermeneutics, and listening, such notions are founded on an awareness of this sort of deep ecological interweaving.

Ecological Mathematics

If the current philosophical and theoretical debates are any indication, the question, What is mathematics?, is an unanswerable one—and fortunately so. Were we able to decide once-and-for-all what mathematics is, we would doubtlessly give in to our modern tendencies of exorcising ambiguity and mechanizing complexity, in effect reducing a dynamic form to a static formula.

A review of the history of mathematics reveals a dynamic and ever-evolving field of inquiry which could never be technologized. To illustrate this point, I describe several "eras" of mathematical history, attempting to show how the era helped to define mathematics and how, in turn, mathematics helped to define the era. Implicit in this analysis is the notion that mathematics is time- and context-dependent. Philip J. Davis and Reuben Hersh, two professional mathematicians, say this on the issue:

> A detemporalized mathematics cannot tell us what mathematics is, why mathematics is true, why it is beautiful, how it comes to be, or why anybody should care a fig about it. But if one places mathematics squarely within human time and experience, it becomes a warm and rich source of possible meanings and action. Its ultimate mystery is never dispelled, yet it is exhibited as one of the prime creations of the human intellect.[6]

I must precede this quick tour through mathematics history by acknowledging my inability to escape the modernist (mathematical?) tendency to abstract, reduce, and impose structure on an amorphous mass of largely unformulated, and far from validated, experience and observation. It is thus that I have identified five *mentalities*[7] in the emergence of mathematics, and will use the terms "Oral," "Pre-Formal," "Formal,"

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[6] Philip J. Davis and Reuben Hersh, *Descartes' Dream: The World According to Mathematics* (Boston, MA: Houghton Mifflin, 1986), 201.

[7] I am using the word "mentality" rather than a temporally situated term (such as "era") because, as should become apparent in the reading, elements of each of the five categories continue to be enacted in some form. I must thus emphasize that the "mentalities" I describe are as much historical eras as they are different ways of thinking about mathematics.

"Hyper-Formal," and "Post-Formal" to refer to them.[8] The centrality of the term "formal" in each of these titles is intended to point toward the relative importance of formality, formalization, and the formulaic in the various conceptualizations.

Of course, mathematics is not and has never been an isolated discipline. The perspectives that are implicit in the field have always spilled into and have always been affected by the perspectives and the developments in other areas of inquiry. For this reason, those readers familiar with the field of Literary Criticism may wish to replace the word "Formalist" with "Structuralist" and, in so-doing, note the relationships between the emergence of current conceptions of mathematics and the evolution of literary interpretation theory. Similarly, some of the relationships between mathematics and postmodern thought will be made more apparent by substituting "Modern" for "Formal."

Mentality 1: Oral Knowledge

The oft heard suggestion that the philosophy of René Descartes gave rise to modernist perspectives is, perhaps, somewhat reductionist and facile. Although he certainly announced a changing of mind-sets, and more than likely contributed to the rise of a new era, Descartes was as much a product of modern thought as he was an instigator.

Walter Ong[9] suggests that the construction of the radical subject (of the sort implied by Descartes' *cogito*), did not begin with the work of Descartes but with the advent of literacy. Starting with the contention that the invention of writing (and, in particular, of the alphabet) separated earlier ways of thinking from modern modes, Ong undertakes to demonstrate how oral societies differ from literate ones. This task is made difficult by the fact the he and his audience are thoroughly inscribed by literary traditions; so constituted, we can at best struggle to imagine how members of an oral culture might think.

Using as his starting point those relics from earlier cultures that have been preserved (albeit in written form), as well as studies of contemporary societies that are predominantly non-literate, Ong argues that knowledge in oral traditions exists only in action.[10] It is thus local, current, practical, and fluid—as necessitated by the fact that ideas exist only in an oral milieu. Put differently, ideas find form in the transient immediacy of sound, a medium which exists only when going out of existence.[11] As

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[8] With the exception of "Oral," these terms are similar to those used by Imre Lakatos in *Proofs and Refutations* (Cambridge, GB: Cambridge University Press, 1976), an insightful and influential analysis of various perspectives on mathematical knowledge. While there are some correspondences to his definitions of the terms, my uses are not intended to parallel his.

[9] Ong, *Orality and Literacy: The Technologizing of the Word*.

[10] I do not mean to imply that oral cultures have no "accumulated knowledge." Quite the contrary, to use a modern "containment" metaphor, their stories and songs serve as repositories of knowledge and wisdom. However, unlike the literate's conception of history and epic tales as essentially static objects (and hence things that are re-read), the oral citizen's stories change with each telling as narrator and audience interact. Histories are not penned (i.e., *constrained*) because they are not penned (i.e., *written*). Put differently, knowledge is not *stored* in the narrative; rather, knowledge is collectively and actively *storied* in the (transformative) recitation of the narrative. It is immediate, negotiated, mutable; it demands that active participation of both teller and listener, implicating all.

[11] To elaborate, we cannot hear a whole word all-at-once. In order to hear "word," for example, the "wo-" has to fade before the "-rd" arrives. By contrast, with written text, words are presented (and recognized) in their entirety.)

such, knowledge resists the objectifying and solidification pressures of written (visual) forms. The oral culture is thus one of activities rather than artifacts.

By consequence, oral cultures are uninterested in definitions; meanings are implicit by usage and through enactment. Where there are gaps in understandings between persons, these are immediately negotiated. In contrast to the literate's location of meaning in language, the meanings of oral traditions are located in contexts and actions. Spoken words come into being in situations (whereas written words tend to be isolated from the setting in which they were recorded). It is thus that persons from oral societies resist—or may simply be unable to—provide definitions for familiar words; for them, words are not items but patterns of acting.

Because of the temporal and contextual embeddedness of all knowledge in oral societies, Ong contends, reasoning does not consist in the abstract logical-deductive modes that we literates associate with rationality. Ong uses the work of Luria,[12] a student of Vygotsky, to illustrate this point. Luria studied some of the non-literate citizens of remote parts of the Soviet Union, noting that these people, for example, identified circles or squares in terms of concrete objects (e.g., plates and mirrors) rather than assigning them abstract names. Further, objects were classified by practical situation rather than in formal categories (e.g., they had difficulty selecting the object that "did not belong" from among a hammer, a saw, a hatchet, and a log, regarding them all as having to do with workmen and wood. Although "tool" was in their vocabularies, it was an object of immediate practical use for them and could thus not serve as a conceptual category). And, perhaps most significantly, each non-literate was virtually unable to speak of him- or herself, modulating self-evaluations into group evaluations. Ong sums up:

> [An] oral culture simply does not deal in such items as geometrical figures, abstract categorization, formally logical reasoning processes, definitions, or even comprehensive descriptions, or articulated self-analysis, all of which derive not simply from thought itself but from text-formed thought.[13]

Edmund Carpenter, in a study of Inuit conceptions and perceptions of "reality," provides support for Ong's conclusions. In particular, Carpenter offers us a glimpse into Inuit senses of "space," arguing that these tend to be framed orally rather than visually. Noting that he is aware of "no example of an Eskimo describing space primarily in visual terms"[14] and that they have no system of linear measurement, he suggests that their spatial apprehension is vague, local, and dynamic—much in contrast to our own senses of well-marked, thoroughly measured, and long-fixed territories. Theirs is a space structured by sound.

The Inuit people in Carpenter's study had no concept of equal portioning (i.e., knowledge of fractions or processes that we would describe in terms of fractions); a universe structured auditorially need not be divided into "equal shares." That is to say that concepts of *ratios*—the term from which "rational" and "reason" are derived—are not part of their oral culture. Furthermore, the counting systems of Inuit peoples typically extended only to five or ten (and, in some cases, only to two or three) before the quantity of "many" was invoked.

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[12] Aleksandr Romanovich Luria, *Cognitive Development: Its Cultural and Social Foundations* (Cambridge, MA: Harvard University Press, 1976).

[13] Ong, *Orality and Literacy: The Technologizing of the Word*, 55.

[14] Edmund Carpenter, *Eskimo Realities* (New York: Holt, Rinehart and Winston, 1973), 37.

At first hearing, statements such as these might be taken to imply that oral societies have no mathematics. In one sense, this is likely true. Mathematics certainly does not exist as an independent discipline in these settings, nor is there a clearly articulated *modus operandi* that can be called "rational" (i.e., deductive). However, as revealed by the poems, the songs, and the stories that are left to us from oral cultures, there was certainly some knowledge of number, statistics, pattern, and meter. (These latter elements, in fact, infuse the oral narratives, serving as mnemonic devices and providing structure for poets and performers.) The essential difference, however, is that numbers and statistics were never divorced from human activity, just as the oral stories were never presented in absence of active listeners. Numbers, in other words, were always adjectives and never nouns. The "mathematics," like the "literature," encompassed, engaged, and implicated both speaker and hearer. More appropriately, perhaps, there was no mathematics (like there was no literature); there was rather the spectral presence of a mode of thinking that might be called "mathematical" and out of which formal mathematical thought eventually arose. It was a mode of thought which, among other qualities, involved the noting and deliberate extending of patterns. Knowledge, in every sense, was situated eco-logically—that is, in the knotted logic of one's *ecos* (dwelling place).

Mentality 2: Pre-Formalist Mathematics

Formal mathematics, then, does not exist in oral cultures, although some form of mathematical thought does. The manner in which the objects of mathematics came into being has thus been a topic of intense and diverse speculation for centuries, with recorded contributions to the discussions dating back beyond the ancient Greeks and Egyptians. Commenting on its possible beginnings, Alfred North Whitehead[15] has conjectured that formal mathematical thought began with the conceptual leap of thinking of seven fishes or seven days to thinking of seven. In this realization of samenesses, the first objects of mathematics—pure numbers—were created.[16] Interestingly, as time passed and as this abstracted notion of number began to permeate human interaction, pure number came to resist "embedding in any human context,"[17] gaining an existential status on par with that of colors and sofas.

Exactly how such abstract notions came to escape their situatedness—how the adjectives became nouns—is an interesting question. Ong suggests that the ability to draw such abstractions is linked to the invention of the written word, a technology which he argues brought about a transformation of human consciousness. According to Ong, writing pushes the *known* into the visual field, detaching it from its author and it audience by assigning it permanence and reducing its mutability. Because of this detachment, the written word can no longer be interrogated. Moreover, through writing, thought becomes a solo (although still social) activity. By writing, the author separates not just descriptor from described, but knower from known and self from other. New senses of subjectivity and autonomy thus arise.

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15 Alfred North Whitehead, *Science and the Modern World* (London: Free Association Books, 1926).

16 While this leap may seem mundane to our modern perceptions, the magnitude of this insight should not be underestimated. Donaldson, in *Human Minds: An Exploration*, and Rudy Rucker, in *Mind Tools: The Five Levels of Mathematical Reality* (Boston, MA: Houghton Mifflin, 1987), discuss this intellectual milestone, both in terms of the difficulty children have in achieving it and in terms of the tremendous conceptual advance that it represents.

17 Donaldson, *Human Minds: An Exploration*, 90.

The written text also enables the reader to work on individual bits—to extract, or abstract, knowledge from contexts and to fragment it into autonomous facts and independent disciplines. In fact, in a very real way, *words* (like numbers) come into existence through the technology of symbolization. They become things with objective status, and thus objects that can be operated upon. Writing also demands that these fragments be presented linearly, and the resulting chains of reasoning mark a profound break with the all-at-once thinking modes of the oral traditions. In a further break with orality, definitions become important as one moves to the visual milieu of writing; lacking the space to interrogate the author, usage must be clear, unambiguous, and uniform.

As a result of these changes in human modes of thought, Ong provocatively suggests, logical-deductive reasoning—which relies on abstraction, linear thought, causal links, fragmentation of ideas, and word-objects (for logic operates on the *logos*—the word)—was a consequence of literacy. He suggests, in fact that formal logic was invented by the Greeks (and other civilizations) soon after they perfected their alphabet:[18]

> We know that formal logic is the invention of Greek culture after it interiorized the technology of alphabetic writing, and so make a permanent part of its noetic resources the kind of thinking that alphabetic writing made possible.[19]

In his own account of mathematics history, Carl Boyer[20] provides support of Ong's contention as he notes that the emergence of formal mathematics has tended to lag some centuries behind the development of literary forms. He also comments that "few subjects depend as heavily on a continuous bookish tradition . . . as does mathematics."[21] He does not, unfortunately, develop the possibility of a relationship between these phenomena.

It is important to emphasize that the association being made here is not between formal mathematical reasoning and language—the latter of which Ong would contend emerged long before logic—but between modern mathematics and literacy. In terms of the shape of our knowledge, this connection represents a shift in traditions from the auditory to the visual. Presented to the eye in the written text, in effect, ideas[22] were abstracted from the lived sonorous realm. They came to have an existence of their own, one that was in many ways superior to the immediately experienced, for the reified forms of these ideas could be applied across experiences.

The transcendent nature of mathematical objects (that is, the way in which abstract mathematical ideas can be applied to situations with no surface similarities) likely contributed to a pervasive conception of mathematics as mystical and magical. Indeed, as Donaldson[23] points out, at some point in our history, mathematicians were

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[18] An interesting side note is provided by Mark Johnson in *Moral Imagination: Implications of Cognitive Science for Ethics* (Chicago, IL: The University of Chicago Press, 1993). He suggests that the individual develops logical thinking when language skills are sufficiently developed to allow her to construct chains of causal thought. In his words, there "exists an intimate connection between life stories and the structures of rationality. These stories are our most basic contact with rational explanation." (p. 179) In other words, Johnson is suggesting that the individual's deductive thinking emerges in a similar manner in which our culture's deductive logic was developed.

[19] Ong, *Orality and Literacy: The Technologizing of the Word*, 52.

[20] Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics, Second Edition* (New York: John Wiley & Sons, Inc. 1991).

[21] Ibid., 280.

[22] "Idea" comes from the Greek *idein*, to see.

[23] Ong, *Orality and Literacy: The Technologizing of the Word.*

thought to be dabbling in "black magic," perhaps as a result of the tremendous predictive capacities of the discipline. (Descartes himself, it appears, actually suppressed many of his mathematical insights, fearing that he might be accused of some form of sorcery.)

Mathematics was thus woven into the spiritual lives of our ancestors. At least since the time of Pythagoras,[24] who proposed that nature is fundamentally mathematical, numbers were seen as the true essence of things; concepts were understood to have emerged from the mind of God; the elegance and "power" of mathematical knowledge hinted at an ultimate "perfection." Mathematical ideas were understood to be the strands from which the universe was woven. One's hold on these threads permitted access to the hidden and true meaning of existence itself. Very often, then, mathematical inquiry was the domain of the priest classes.

Mathematical ideas were thus entangled with all aspects of existence and all areas of knowledge. They were not something imposed on, but qualities inherent in, all things. Correspondingly, as George Steiner[25] suggests, all mathematical concepts could be represented in conventional linguistic terms—that is, a concept was always interpretable against one's everyday experience. According to Steiner, and as will be developed later, this perspective on mathematics persisted until the time of Galileo, Descartes, and Newton.

The tight links with daily experience and familiar language contributed to a conception of mathematical *truths* as "forms" or "unchanging aspects" that governed the world. In this conception, mathematics was rigorous and definite, as epitomized by Euclid's geometry. However, far from being a collection of theorems or established truths—that is, a distinct discipline or body of knowledge—mathematics was powerfully regarded as a particular mode of inquiry that was, for the most part, as tightly linked to the arts of divination as to other areas of inquiry. As Heidegger explained, mathematics (or, more accurately, the *mathematical*) was "the fundamental condition for the proper possibility of knowing" which involved an awareness of "the fundamental presuppositions of all knowledge and the position we take on such knowledge."[26] Stated otherwise, the *mathematical* was regarded as a particular and disciplined approach to thinking that began with the statement of fundamental propositions (the *axioms*, arrived at by consensus) and, through the applications of particular rules (the *logic*), the deduction of further truths. (It is thus that Thales, to whom the fist deductive proof is often attributed, is hailed as the first true mathematician.) The validity of the results, assuming no rules were violated, was assured by the rigor of the derivation process—not by some external measure.

This is not to say that mathematical thought disregarded experience. On the contrary, the purpose of such analysis was to enrich those experiences, locating them in the web of creation by referencing all postulates to their bases in axioms. Nor should we make the mistake of associating all of pre-formalist mathematics with the logico-deductive structures of the *geometria*. While the Greeks were exceptional in their concern with formal knowledge, their predecessors, contemporaries, and immediate successors tended to focus on the practical aspects of mathematical inquiry, mixing derivation and

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[24] Pythagoras is often given credit for coining the term "mathematics" ("that which is learned"), a word that he used to describe his intellectual activities. See Boyer and Merzbach, *A History of Mathematics, Second Edition*.

[25] George Steiner, *Language and Silence* (New York: Atheneum, 1967).

[26] Heidegger, *Basic Writings*, 254.

approximation with little apparent concern for what we would consider an inconsistent application of principles.

Nevertheless, contrasting the pre-formal conception of mathematics with the sense of "mathematical" that exists in oral traditions, one notes a dramatic departure. Still shared are the senses of order and pattern; however, with the influence of symbolization, the mathematical has acquired an almost magical capacity to pull objects of reason to material form.

Mentality 3: Formalist Mathematics

As mentioned, an important transition in mathematics was initiated in the time of Descartes and Newton. As Steiner explains, mathematical methods were turned upon themselves, producing a realm of knowledge that could no longer be meaningfully reduced to or captured by conventional language. As might be illustrated with the example of Newton's *Calculus*, mathematics broke from the everyday and ceased to be understandable or representable in terms of immediate experience. This "break"—which amounted to a removal of intuitive restraints—had a number of important consequences, among them a tremendous surge in research and a corresponding increase in mathematical knowledge. As such, mathematics ceased to be regarded merely as a mode of reasoning and became a distinct discipline, separated from other areas of inquiry. That is to say, the *mathematical* began to be overshadowed by the *mathematics*—i.e., the mode of thinking was in some ways hidden by the body of knowledge that it spawned.

At the same time, and largely through the work of Galileo, Descartes, and Newton, science was mathematized through the application of the rigorous guidelines for mathematical inquiry to the study of diverse physical phenomena. These events mark the (formal) beginning of the *modern* era, and it is thus not surprising that the person most often credited (or blamed) for the rise of modernity, Descartes himself, was a mathematician. Reacting to the current state of knowledge, which he considered to be a mixture of fact and fancy,

> [Descartes] had been the first to embark upon a programme to establish a firm foundation for human knowledge of the world and had singled out mathematics as the only reliable route to unimpeachable knowledge.[27]

In other words, Descartes called for nothing less than "the primacy of world mathematization."[28] This transitional period is thus marked by three critical shifts in thinking: mathematical ideas came to be seen as something apart from human experience; the mathematical and the scientific were given identities distinct from the religious, the magical, and the spiritual; and mathematics acquired the status of *the* model of reasoning for a modern era. The overarching goal of this movement—or, perhaps more appropriately, its net effect—was prophesied in 1637 by Descartes in his *Discourse on Methods*:

> [My discoveries] have satisfied me that it is possible to reach knowledge that will be of much *utility* in this life; and that instead of the speculative philosophy now taught in the schools we can find a practical one, by which, knowing the nature and behavior of fire, water, air, stars, the heavens, and all the bodies which

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[27] Barrow, *Pi in the Sky: Counting, Thinking, and Being*, 127.

[28] Davis and Hersh, *Descartes' Dream: The World According to Mathematics*, 8.

> surround us, as well as we now understand the different skills of our workers, we can *employ these entities* for all the purposes for which they are suited, and so *make ourselves masters and possessors of nature*.[29]

All sense of our ecological-embeddedness has been cast aside in this formulation. Similarly, the experienced, phenomenal world is rendered suspect, fallible, and unreliable. It is thus that mathematics displaced religion, history, and narrative as legitimate routes to knowledge and became "the unifying glue of a rationalized world."[30] Descartes emphases on the detached, technical, and utilitarian qualities of mathematics contributed to an emerging conception of the universe as a machine—and our relationship to that machine, as revealed in Descartes' own words, came to be expressed in the language of control and dominance via mastery of the underlying mathematics.

As is evidenced by the preeminent place of mathematics within modern universities and government agencies, Descartes' dream of world mathematization has largely come to pass. Mathematics, primarily through our science and our technology, has come to permeate our existence.[31] The result has been that, adapting Gadamer's worlds on *history*, "In fact [mathematics] does not belong to us, we belong to it."[32]

It is this formalist view of the discipline and the consequent pervasiveness of mathematics which have drawn such intense criticism from various theoretical perspectives. Postmodernist thinkers[33] have identified mathematics as one of the "grand narratives" which have compelled us, the victims of modernity, to suppress our own personal narratives. The consequence has been a feeling of profound personal displacement broadly inflicted upon the citizenry of Western nations. Ecological philosophers[34] suggest that the equating of mathematics and rationality has caused us to ask only "Can we do it?" rather than the more ecologically sound "Should we do it?" The result has been the near-sighted, and often disastrous, application of our mathematized scientific knowledge. This form of "conventional rationality" has not only eclipsed other forms of reason, it has allowed greed to replace the tradition of wisdom. Various feminist scholars[35] argue that, far from being the "neutral" discipline it is held to be, mathematics is culturally-, socially-, and gender-biased. Our modern fetishing of mathematical knowledge has thus contributed greatly to social inequities. Even mathematicians have joined in the attack on formalist mathematics. Davis and Hersh,[36] for instance, discuss the consequences of the fulfillment of Descartes' dream in their survey of the contexts in which and the phenomena onto which mathematics has been applied—including the inappropriate application of its principles to various social situations and the insidious use of its insights in the enabling of our military culture. David Michael Levin powerfully

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29 René Descartes, *Discourse on Method and Meditations on First Philosophy* (Indianapolis, IN: Hackett Publishing Company, Inc., 1993). (emphasis added).

30 Davis and Hersh, *Descartes' Dream: The World According to Mathematics*, 12.

31 Accessible and insightful accounts of this "world mathematization" are offered by Davis and Hersh in *The Mathematical Experience* and in *Descartes' Dream: The World According to Mathematics*, and by Barrow, in *Pi in the Sky: Counting, Thinking, and Being*.

32 Gadamer, *Truth and Method*, 276.

33 e.g., Jean-François Lyotard, *The Postmodern Condition: A Report on Knowledge* (Minneapolis, MN: Minnesota Press, 1984).

34 e.g., Bookchin, *The Philosophy of Social Ecology: Essays on Dialectical Naturalism*.

35 e.g., Valerie Walkerdine, *The Mastery of Reason: Cognitive Development and the Production of Rationality* (London: Routledge, 1988).

36 Davis and Hersh, *Descartes' Dream: The World According to Mathematics*.

sums up these critiques as he points to "the terrible violence, the subtle repression of difference and otherness, hidden within the 'benevolent universality' of Reason."[37]

Mentality 4: Hyper-Formalist Mathematics

Despite the privileged position of mathematics within modern society, its foundations have long been questioned. The seemingly endless dispute on the issue of the criteria for a valid proof, for example, suggests that mathematics reputation for certainty may be no more than an illusion.

In particular, the emergence of non-Euclidian geometries nearly two centuries ago signaled an important transition in mathematics as it became clear that one could question and manipulate the very foundations of mathematical systems. The full consequences of this development were not felt until the beginning of this century when prominent mathematicians David Hilbert, Albert North Whitehead, and Bertrand Russell set out to reconstruct mathematics as a strictly formal system. In effect, one of their goals was to articulate the inner consistency (and hence the independent nature) of mathematics by divorcing it from the experiential world[38]—a project which, as noted above, had its origins in the work of Newton and his contemporaries.

While their purpose was hardly to wring the meaningfulness out of mathematics, the effort to formulate a fully consistent system served to distance the discipline even further from its already obscure associations with the realms of the phenomenal, the superstitious, and the mystical. This conception was bolstered by an emerging division within the field itself, as "applied" began to be distinguished from "theoretical" mathematics. To this point, mathematics had been "the servant of science and technology,"[39] with efforts centering on such applications as land surveying and astronomical calculations. In the last century, however, the inferior status of mathematics was pushed aside as mathematicians "invented their own problems and played gratuitous games whose rules they made up themselves."[40] Hubert Reeves suggests that this led to an astonishing discovery:

> The questions posed by science and technology represented only a tiny fraction of the problems that could be formulated. Most theories devised by mathematicians . . . *had no application in reality*. In no way did they describe the world around us. Their axioms did not correspond to nature as we know it. They existed only for the pleasure they gave to the mathematicians who invented them. Their sole justification was their own internal coherence.[41]

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[37] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 33.

[38] It is important to note that these mathematicians were not attempting to render mathematics *devoid* of meaning. Rather, their aim was to relieve mathematical principles of the excess weight of associated (meaningful) experience and, in so-doing, to allow mathematics to become more powerful. At the same time, it was felt, the potential for new and more profound mathematical insight would be enabled since much of the obscuring—and, to the formalist perspective, irrelevant—details could be disregarded. Hilbert's goal, in fact, might be expressed in terms of simplifying the process of proof. He sought a "meta-mathematics" of sorts: a means of checking a proof by stepping outside it, thus providing verification without having to return to first principles.

[39] Reeves, *Malicorne: Earthly Reflections of an Astrophysicist*, 27.

[40] Ibid.

[41] Ibid., 27. (original emphasis)

One can see the emerging separation between "discovered" and "invented" mathematics here. Once not an issue—for mathematics was believed to permeate the universe—the tension began to deepen the chasm between experience and mathematical interpretation. The effect—on popular (public) opinion, at least—was to promote a conception of the discipline that had very little to do with the "real" world. In this formulation, mathematics became the quintessential "grand narrative," transcending not only experience, but human existence.

During this time, the mathematization of scholarly study crept beyond the physical sciences and into such realms as economics, politics, language, and law. This movement was, as Davis and Hersh put it, "based on the questionable assumption that problems in these areas can be solved by quantification and computation."[42] The extent to which a discipline relied upon mathematics came to be taken as a measure of its rigor and, hence, its relevance. Moving away from the conception of an "established body of knowledge," mathematics came to be defined as something that mathematicians do. Nevertheless, perhaps as a result of its growing influence, beliefs in its neutrality, its pristine structure, and its objective truthfulness were further entrenched.

Mentality 5: Post-Formalist Mathematics

In the 1920s, mathematician Kurt Gödel demonstrated that Hilbert's hope of devising a decision algorithm for all of mathematics and the Whitehead-Russell project of deducing all of mathematics from the axioms of logic were unrealizable by proving that any sufficiently complex mathematical system is necessarily incomplete. The hyper-modern project, in effect, came crashing to the ground as the issue of the nature of mathematical knowledge was torn open. For the first time in the modernist/formalist era, absolutist beliefs in mathematical knowledge began to be eclipsed by more fallibilist accounts which acknowledged the social dynamic of knowledge generation.

Discussions in the area have come a great distance since Gödel's earth-shaking pronouncement, with important contributions coming from the likes of Karl Popper,[43] Thomas Kuhn,[44] and Imre Lakatos.[45] More recently, a few mathematicians have dared to suggest that mathematics might more appropriately be considered as one of the *humanities*.[46] Inherent in such a conception is the suggestion that mathematics, as a human construction, is fallible, ill-structured, and implicitly biased. More importantly, mathematics so-conceived tells us more about ourselves than about the universe which was once believed to be written in mathematical characters. In other words, concerns for the qualities of the *mathematical*, versus the objects of *mathematics*, have re-emerged.

Other recent developments in mathematics have served to further debunk realist and formalist perspectives (which, in spite of these events, continue to have a pervasive presence in the discipline). Notably the tremendous advances in computer technology, coupled with the increased availability of such technology, have had profound effects on both the approaches one might take to mathematical investigation and the ways one might go about proving particular ideas.[47] Two events stand out in this regard. First, in 1976, a

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[42] Davis and Hersh, *Descartes' Dream: The World According to Mathematics*, 16.

[43] Karl Popper, *The Logic of Scientific Discovery* (London: Hutchinson, 1959).

[44] Thomas Kuhn, *The Structure of Scientific Revolutions* (Chicago, IL: University of Chicago Press, 1962).

[45] Lakatos, *Proofs and Refutations*.

[46] Davis and Hersh, *The Mathematical Experience*.

[47] See John Horgan's "The Death of Proof," *Scientific American*, October, 1993, 92-103.

group of mathematicians proved the century-old four-color conjecture[48] by combining graph theory and sophisticated computing. Second, the dynamic and increasingly popular new field of non-linear dynamics (Chaos Theory), sparked by an unexpected result from a computer-simulated weather system, opened the possibility of using computers as an investigative tool in a more "experimental" and tentative approach to mathematical research.[49] Lynn Arthur Steen sums up recent developments in mathematics as follows:

> Not since the time of Newton has mathematics changed as much as it has in recent years. Motivated in large part by the introduction of computers, the nature and practice of mathematics have been fundamentally transformed by new concepts, tools, applications, and methods. Like the telescope of Galileo's era that enabled the Newtonian revolution, today's computer challenges traditional views and forces re-examination of deeply held values. As it did three centuries ago in the transition from Euclidian proofs to Newtonian analysis, mathematics is undergoing a fundamental reorientation of procedural paradigms.[50]

These developments, while contributing to a renewed public interest in mathematics, have hardly been welcomed with open arms within the discipline. Many feel that they do not represent mathematics at all; others question the trustworthiness of programming or the accuracy of digital computers. There are continued calls for the rigorous paper-and-pencil proof—even though, with specialization and complexity both on the rise, the task of validating such proofs without technological aid is becoming a significant challenge.

While the debate rages on as to whether Chaos Theory is indeed mathematics, this area of study has served to demonstrate that mathematical knowledge is not pre-existent; nor does it exist in any one of us. Rather, it emerges from our actions in the world and from our interactions with one another. The recognition of the intersubjective nature of mathematical inquiry is suggested by John Barrow:

> [An] intriguing aspect of mathematics that seems to distinguish it from the arts . . . is the extent to which mathematicians . . . collaborate in their work. . . . [In] mathematics the collaborative process goes much deeper to *entwine* the *authors* in a process . . . by which they are able to produce a result that could not have been half-reached by one of them.[51]

This statement is worthy of further analysis, more because of what is suggested than because of what is explicitly stated. The implication is that the modernist and formalist orientation to the discovery of truth has given way to a sense of creativity. The mathematician is an *author*—not a theorist, scientist, or explorer—who is *entwined* with her co-authors. The myth of the isolated mathematician (and of isolated mathematics) is thus also put to question. We have moved to an awareness that mathematical meaning,

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[48] The Four-Color Conjecture: "any planar map can be coloured using at most 4 colors in such a way that no two adjacent areas are on the same color."—E. J. Borowski and J. M. Borwein, *Harper Collins Dictionary of Mathematics* (New York: HarperPerrenial, 1991), 227.

[49] A thorough account of the emergence of this branch of mathematics is given by James Gleick in *Chaos: Making a New Science* (New York: Penguin Books, 1987).

[50] Lynn Arthur Steen, "Pattern," in *On the Shoulders of Giants: New Approaches to Numeracy*, ed. Lynn Arthur Steen (Washington, DC: National Academy Press, 1990), 7.

[51] Barrow, *Pi in the Sky: Counting, Thinking, and Being*, 267-268. (emphasis added)

like scientific meaning, "derives from both social interaction and interaction with the physical world."[52]

In a sense, then, the post-formalist era involves a re-claiming—an *informed* re-claiming, that is—of past perspectives on knowledge. It offers the chance to recover some of the pre-formalist wonder associated with mathematics—or more particularly, the *mathematical*—as it re-situates us in a world of actions, interactions, and interpretations. Mathematics is once again understood to emerge as we seek to understand the transcendent complexity of our universe. As such, it provides us with a sense of the "pattern which connects"[53] us to our worlds.

In conceiving of mathematics in this way, it becomes apparent that the popular understanding of a "mathematical concept" is in error. In the movement toward a formalist mathematics, our actions within the world (such as counting, comparing, and pattern noticing) have become solidified into objects (like number, size, and order) whose existences have somehow become conceptually independent of—and even prior to—the actions, experiences, or insights that brought them forward. It also seems that mathematics, rather than representing a distinct mode of reasoning, is entangled with other forms of rational thought, including narrative and metaphorical. Once again, mathematical understanding is closely aligned with one's experiences and—in opposition to the formalist tendency toward the dissociation of concepts from their originary experiences—personal meanings are considered to be fundamental to understanding.

Discovery or Creation?

Even though recent events have pushed the issue of the nature of mathematical knowledge to the foreground, it seems that Gödel's Theorem, chaos theory, computer-assisted proofs, and other developments have served to complicate rather than to simplify the debate. In the final analysis, the question remains unanswered: Is mathematics discovered or created? Or, more formally, do mathematical "objects" have a real existence or is mathematics essentially a mental activity?[54]

Certainly the current discussions in mathematics education favor the latter perspective. But many participants continue to find the idea that mathematics is something we develop (and subsequently impose on an unsuspecting universe) to be as untenable as the opposing Realist perspective that mathematical truths, quite literally, are hiding in the bushes awaiting discovery. How, for example, can we account for the tremendous descriptive capacities of mathematical ideas if they are only mental activities?[55]

That we feel we must believe one way or the other, I think, is evidence of our inability to escape the modern mind-set that we are essentially independent and isolated entities. Not only are we distinct from one another, we are set apart from the universe. To maintain that mathematics is extracted from the world—i.e., discovered—is to subtract

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52 Davis and Hersh, *Descartes' Dream: The World According to Mathematics*, 86.

53 This is one of Gregory Bateson's provocative phrases.

54 Not surprisingly, a diverse collection of theories and "philosophies" which purport to provide viable accounts of mathematical knowledge have arisen recently. Paul Ernest, in *The Philosophy of Mathematics Education* (London: The Falmer Press, 1991), provides a concise overview of many of these perspectives.

55 For a more thorough articulation of this question, see Richard W. Hamming, "The unreasonable effectiveness of mathematics," *American Mathematical Monthly* 87 (February, 1980): 81-90.

the creative genius of the mathematical community and to deny our role in shaping the world in which we exist. To argue the contrary point, that mathematics is created, is to ignore the world which provides the occasions for and the constraints upon our own thought. In its extreme form, this argument leads to the conclusion that human thought is somehow unnatural—not of nature. Our ecological situation cautions us against maintaining this naive belief.

Much of our difficulty in addressing this issue, I believe, arises from the modern proclivity to objectify knowledge and, in so-doing, to separate it from ourselves. Even with the movement from Realist to more subjective accounts of knowledge, for example, it seems that we continue to cling to an image of truth as some sort of "cloud"—albeit placed on a social rather than a platonic plane. There is, however, the possibility of avoiding this tendency, but it requires that we look at mathematics differently. More appropriately, perhaps, it requires that we cease to privilege *looking* (trying to find an object) and begin to *listen* more attentively (locating ourselves within dynamic and complex historical and social situations).

By modifying "mathematics" with the term "ecological," a different sense of *knowledge* is implied—a knowledge that is neither uncovered nor invented, but which emerges from—that is, it is *enacted* through—the history of our collective participation in a dynamic and responsive world. In this sense, mathematical knowledge is analogous to the subject matter of a conversation[56]: its nature, its structure, its conclusions can never be anticipated, let alone captured or fixed. Efforts to impose such analyses are necessarily reductionist, disregarding the myriad of complex (and often subtle) factors that help to shape it. Like the conversation, "what mathematics is" depends upon the interplay of the era, the culture, the specific setting, the particular participants.

Far from merely *representing* the universe (thus setting us apart from it), our mathematics *presents*[57] the rhythms of the planet and the patterns that are repeated in all forms and at all levels of life. It does not *reduce* the universe, but places us in conversation with it, hinting at the complex orders and the tangled relationships which inevitably exceed our attempts to understand and surpass our efforts to control. Conversely, our mathematics also *presents* us (i.e., "makes us present") in these harmonies, enacting not the modernist separation, alienation, and exile from the natural world, but an attunement to the pulse of the planet.

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[56] In proposing this analogy, I must also explain that the idea is not intended to be associated with the notion that mathematics is a *language*—an idea by which mathematics is posited as a sort of third object that stands between persons and which mediates their relationships. It is this sort of definition or analogy—that is, one which denies the dynamic complexity of mathematical inquiry—that underlies technical and fragmented perspectives of the discipline. If mathematics is a thing, that it must either be a discovered or a created thing.

Two texts in particular have contributed to this formulation. David Pimm's *Speaking Mathematically* (London: Routledge and Kegan Paul, 1987) includes a brief argument against the notion that mathematics is a language. Roger Penrose, in *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics* (New York: Vintage, 1989), asserts that the popular claim that mathematical reasoning is the same as linguistic reasoning is wrong. Quite apart from always enabling mathematical thought, Penrose suggests that language often serves to constrain it.

[57] This idea is borrowed from Gadamer's description of the work of art—an idea that I return to in the next chapter.

Moreover, in the enactivist frame, mathematics can never be understood merely in terms of the actions[58] of the subject or in terms of the qualities of the object. Rather, mathematics emerges in the mutually specifying dynamics between the activities of the subject and the reactions of the object. Neither are pre-existent; rather, they arise only in action. Further, both are altered as a result of the action. In many ways, then, a post-formalist mathematics is a return to the situated, temporal mathematics of the oral traditions.

To make the point in a different way, we must guard against the modern tendency to posit mathematics (and all knowledge) as a sort of "third thing" that spans the chasm between isolated knowers and the inaccessible known world. The troublesome feature of this formulation is that cognizing agent and world are cast as two distinct categories; as Merleau-Ponty has pointed out, it is a conception that has forgotten the body.

The body is simultaneously of oneself and of the world. My body separates me from other bodies at the same time that it places me in relationship with them. My body is shaped by the world that it participates in shaping. There are not three things, then—mind, world, and, between them, a body. Rather, mind and world are rendered inseparable by the body.

Merleau-Ponty's insight into our "double-embodiment" appears to be closely aligned with conceptions of our intercorporeality that are emerging from such disciplines as economics, physics, and political science.[59] Briefly, there is a growing awareness that our (seemingly) separate bodies are able to act in ways that are not merely coordinated, but which give rise to structures that transcend each of us. The same sort of phenomenon occurs as the subsystems that comprise each of our bodies come together to form more complex unities. Collectively, particular properties, qualities, and patterns emerge that cannot be possessed individually (nor can they be anticipated based on one's knowledge of the individual components). On the social (interpersonal) level, these properties and patterns of acting can be thought of as our collective knowledge.

An essential tenet of enactivism is that knowledge (individual or collective) is the same as action. That is to say that "what we know" is revealed in "what we do"; our knowledge defines and is made manifest in our actions. One might thus draw an analogy between one's physical body and a social group's body of knowledge. In neither case are there three things. Just as mind and world are made inseparable by the body, knowers and known world are brought together in a body of knowledge. Similarly, just as my changing body is the locus of my personal identity—simultaneously separating me from and situating me in the world—so our dynamic knowledge is the locus of our collective identity—affording an integrity which simultaneously distinguishes us from a background while placing us in communion with that background. Our body of knowledge is also an historical structure, a point that is effectively made by adapting Anthony Kerby's description of the physical body:

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[58] The term "action," here and elsewhere in this writing, is intended to include both external (visible) and internal (invisible) actions. (That I feel the need to mention this point might be interpreted as an indication of the extent to which we privilege the visual over the auditory. In a visual frame we are far more likely to consider thought as inactive.)

[59] See M. Mitchell Waltrop, *Complexity: The Emerging Science at the edge of Order and Chaos* (New York: Simon & Schuster, 1992).

> [A] body [of knowledge] must be seen as the enduring locus to which a [cultural] life history accrues, and hence to which the character of that history is indissolubly associated.[60]

Our body of knowledge—that is, our established and mutable patterns of acting—can thus be thought of as our collective self: co-emergent with, inhabiting, and en-habited by the world.

Our mathematical knowledge, like our language, our literature, and our art, is neither "out there" nor "in here," but exists and consists in our acting. As such, the character of our knowledge changes with every action, as do the characters of the agent and of the world. Varela, Thompson, and Rosch refer to this phenomenon as the "fundamental circularity of being," whereby we are prompted to recognize that the universe changes with an event as mundane as a shifting thought. That thought is not merely located in the universe, it is part of the universe. In other words, just as the individual's cognition can be likened unto a species' emergence, so can the patterns of acting (that we refer to as our collective knowledge) be understood in terms of post-Darwinian evolutionary processes.

What is being suggested here is that we cannot make tidy distinctions between individual and collective or among knowledge, cognition and identity. All subsume and are subsumed in action and interaction.

A New Question

In essence, then, the enactivist challenge is not to offer a new answer to the question, What is mathematics? That issue, they argue, can never be fully resolved because we cannot extract our dynamic selves or worlds from our mathematics. It might be more appropriate, in fact, to reverse the common sensical notion that we produce (whether via creation or discovery) our mathematics, suggesting instead that our mathematics produces us. Indeed, mathematics so permeates our culture and so frames our thinking that we can make little sense of persons and cultures that do not share similar traditions.

The more important question thus becomes, What are *we* that we might know mathematics?[61] And herein lies an important difference between enactivist and modernist perspectives: in the move to embrace the complexities of existence, the focus shifts toward understanding *being* and away from questions of knowledge and validity. Put differently, priority is given to ontological concerns rather than epistemological, thus reversing Descartes *cogito*. The implications are profound, extending culture-wide to encompass virtually every niche of human endeavor—for, in our modern age, little has been untouched by formalist perspectives on knowledge and the divisive and destructive patterns of acting that are supported by such perspectives. This issue will be explored further in the next chapter when the issue of formal education is addressed.

But before leaving this section, I wish to state in clearer terms that, by invoking ecological theories, I am not attempting to establish a new ground for mathematical knowledge. Rather, I am trying to embrace the "groundlessness" that has emerged in all

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[60] Anthony Paul Kerby, *Narrative and the Self* (Bloomington, IN: Indiana University Press, 1991), 111.

[61] Adapted from Warren McCulloch's, "What is number that man may know it, and a man that be may know number?" in *Embodiments of Mind* (Cambridge, MA: MIT Press, 1963), 1.

areas of academic inquiry, aligning myself with Varela et al. who contend that we must work toward a "planetary world." In their words,

> the solution for the sense of nihilistic alienation is our culture is not to try to find a new ground; it is to find a disciplined and genuine means to pursue groundlessness. Because of the preeminent place science occupies in our culture, science must be involved in this pursuit.[62]

In the educative endeavor, then, we must seek to cure what Max Horkheimer call the "disease of reason": "The disease of reason is that reason was born from man's urge to dominate nature; and 'recovery' depends on insight into the nature of the original disease, not a cure of the latest symptoms."[63]

The last word in this exploration of mathematics I give to two mathematicians, Philip J. Davis and Reuben Hersh:

> We should never forget that a stroll in the woods or a deep conversation with a new or old friend are beyond mathematics, And then, when we go back to our jobs as administrators, teachers, or whatever, let us still remember that numbers are only the shadow, that life is the reality.[64]

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[62] Varela et al., *The Embodied Mind*, 253.
[63] Max Horkheimer, *Eclipse of Reason* (New York: Continuum-Seabury, 1974), 176.
[64] Davis and Hersh, *Descartes' Dream: The World According to Mathematics*, 98.

Section B

Mathematics Curriculum

> In order for curriculum to provide the moral, epistemological, and social situations that allow persons to come to form, it must provide the ground for their action rather than their acquiescence. It must be submitted to their reform, be accessible to their response. . . . Curriculum is a moving form.
> — Madeleine Grumet[65]

I recall visiting the bedroom of my niece a few months before she began grade one. There she introduced me to her horses—a large and varied assemblage of playthings, porcelain, and pictures.

It was immediately clear to me that this sprawl of things had prompted her to count, to assemble, to order, to compare. As I followed her movements through the room, I became aware of the subtle and exquisite patterns in her arrangements. Her interest lay—to my hearing—not in the images or in the physical presence of the horses, but in the play that they invited.

Today my niece is in grade three. Today she hates "math." And she's afraid of it.

It's not surprising. She can't do it. From the instant her teacher tells her and her classmates to begin the *Mad Minute*[66] to the sound of the timer's bell, she is confronted with the menacing reality of her incompetence.

Which is not to say she can't do the adding and subtractings. Given enough time (and a table under which to hide her still active hands) she can produce answers without error.

But to her, that's not math. In her math class, fingers and the symbols on the page are only coincidentally related in the word "digits." The work of her hands and the arithmetic manipulations are worlds apart. The math, to her, is in the insanity of the *Mad Minute*. It is a work of the mind that demands the suppression of the body.

Thankfully, she still plays with her horses. And she'll still talk about them with anyone willing to listen.

The topic of this section is curriculum. It is about the business of sifting through what is known and selecting those aspects of our knowledge that are most important to the members and the maintenance of our society. Curriculum-makers thus locate themselves between the culture's established knowledge and the individual's emergent knowings.

More specifically, this section is both a challenge to modern outcome-based perspectives on curriculum and an introduction to what may be the starting place for an alternative. It is, in effect, a call to discard the orientation to mathematical knowledge underlying and promoted by the *Mad Minute* and to seek our curriculum amid the chaos of a child's collection of horses.

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[65] Madeleine Grumet, *Bitter Milk: Women and Teaching* (Amherst, MA: The University of Massachusetts Press, 1988), 172.

[66] A "Mad Minute" is a page of computation questions. Students are given one minute to complete as many of the questions as possible.

Conceptions of Curriculum

The term "curriculum" has been subject to a range of interpretations which share little beyond an acknowledgment that curriculum has something to do with what happens in schools. I will begin this discussion by tracing out perhaps the most prominent orientation, one which finds its roots in the work of Franklin Bobbitt.[67]

Bobbitt began from understandings of society and schooling as essentially static entities, insisting that curriculum development should begin with scientific determinations of that knowledge and those qualities that are necessary for adult life. These elements could then be dissected into teachable bits by curriculum makers. It is thus that, for him, schooling amounted to a preparation for life; it was not part of life itself.

In terms of current beliefs and practices surrounding curriculum, the sorts of ideas announced by Bobbitt figure prominently. This point is well-illustrated in the survey of conventional perspectives on curriculum that has been prepared by William Schubert.[68] Among the more prominent of current orientations, Schubert identifies and elaborates upon the following: curriculum as *content*, as *subject matter*, as *a program of planned activities*, as *intended learning outcomes*, as *cultural reproduction*, and as *discrete tasks and concepts*. Common to each of these orientations are a desire to predetermine what is valuable to know and a belief in the possibility of controlling learning outcomes once the topics of study are selected.

Put differently, each of these perspectives is predicated on the assumption that it is possible for the contents of a curriculum to have a transcendent validity—one which, for all intents and purposes, is independent of the era, the culture, the classroom, the teacher, and the learners. Such an assumption arises from the modern notion that the world is pre-given and objectively knowable—a conception that supports curriculum developers' goals to identify knowledge objectives that reflect that world and to organize those objectives in ways that are suited to the linear and tiered structure of the schooling system.

Conspicuously absent in this formulation is a reference to the learner, and so, not surprisingly, there are those who argue for a more "student-centered" curriculum. Rejecting the stifling effects of imposed formal structures and ostensibly value-free "facts," these student-centered approaches concentrate on personal expression, authority, autonomy, and self-image.

As might be expected, then, an objectivist-subjectivist tension figures prominently in much of current discussion the field of curriculum. Supporters of the a knowledge-centered perspective worry that the child-centered approach is relativistic and potentially solipsistic. Proponents of a more child-centered curriculum criticize their adversaries' project as being de-humanizing and oppressive. This "world versus child" conflict was first announced in Dewey's *The Child and the Curriculum*[69] and it has since become a foundational issue in curriculum studies.

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[67] Franklin Bobbitt, *The Curriculum* (Boston, MA: Houghton Mifflin, 1918); Franklin Bobbitt, *How to Make a Curriculum* (Boston, MA: Houghton Mifflin, 1924).

[68] William H. Schubert, *Curriculum: Perspective, Paradigm, and Possibility* (New York: Macmillan Publishing Company, 1986).

[69] John Dewey, *The Child and the Curriculum* (Chicago, IL: The University of Chicago Press, 1956 [1902]).

Exactly which side of the debate seems to be "winning" is easily established. One need only step inside a typical mathematics classroom and make note of the teacher's position at the head of the room, the program of studies' place in the centre of his desk, and the standardized textbooks located in front of each learner. Student authority and self-image are not priorities in today's math class.

What must be borne in mind, however, is that both objectivist and subjectivist perspectives on curriculum are founded on the belief, à la Bobbitt, that those qualities most critical for successful living can be identified and taught. In terms of the nature of mathematics in this formulation, there is a predictable emphasis on the utilitarian and mechanical qualities of the subject matter. That is, for the most part, mathematics is valued for the very qualities that serve to crystallize the discipline into a completed and static hierarchical structure of absolute concepts and rigid procedures—a resource to be mined and exploited, as it were. In this conception, understanding is reduced to rightness and wrongness and doing math is made equivalent to applying memorized rules.

A Critique

The difficulty with the development of mathematics curricula, I would like to argue, springs from two sources: a narrow definition of curriculum and an a naive understanding of the nature of mathematical knowledge. Having already touched on the latter issue, I will focus here on alternatives to the predominant instrumental and prescriptive orientation to curriculum (predominant, that is, among those agencies responsible for the creation of curriculum documents).

Reflecting the mode of thinking that underlies Bobbitt's writing, "curriculum" is generally understood to refer to those mandated programs of study that are to be offered in our schools. As already noted, for the most part, such curricula take on the physical form of an ordered list of objectives to be met over a period of study and, in the case of mathematics at least, to be assessed through a battery of regularly scheduled standardized examinations. In this incarnation, a curriculum document inevitably and necessarily is comprised of bits of already established facts that are to be passed from one generation to the next.

An immediate difficulty arises, however, when one considers the nature of the well from which the "facts" are drawn, for what seems to be forgotten in the construction of such impositional curricula is that, in Jerome Bruner's terms, "a culture is constantly in process of being recreated as it is interpreted and renegotiated by its members."[70] Unfortunately, like the static appearing glacier, the transitional rate of a culture's collective knowledge has traditionally been so much slower than that of an individual within that culture that this knowledge tends to be "taken for granted"; it is the given that precedes the preparation of curriculum documents. Losing sight of its movement and the mass that lies behind it, we have tended to excavate bits from the front end of this glacier and to offer them as a reasonable representation of the remainder.

As such, modern curriculum has forgotten its past. Moreover, in our efforts to distinguish one discipline from another and one concept from another, our curricula have come to embody the modern ideals of fragmentation and isolation. In consequence, again with reference to school mathematics, the subject matter has come to be regarded as

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[70] Bruner, *Actual Minds, Possible Worlds*, 123.

having little to do with the "real world" and as bearing an even more tenuous relationship to the lived experience of learners.

A number of critical commentaries on this orientation to mathematics curricula have been offered from a range of perspectives. Valerie Walkerdine offers a feminist critique of the pre-eminent place assigned to mathematics within the modern curriculum—a practice which she regards at the foundation of the "bourgeois and patriarchal rule of science, it is indeed inscribed with domination."[71] Social ecologist Murray Bookchin[72] comments on the devastating cultural consequences of our privileging of an ostensibly neutral "conventional reason" modeled after mathematical thought. David Orr,[73] an environmentalist, discusses the contribution of our near-sighted application of mathematized scientific knowledge to the destruction of our ecosystem. A host of critical theorists have echoed and elaborated upon these critiques, all arriving at a similar conclusion: that in our teaching of mathematics, we are maintaining a series of gender-, racial-, class- and cultural barriers, some of which are a consequence of teaching methods, but much of which can be directly traced to the discipline of mathematics itself.

These thinkers are presenting quite a different message from those few theorists and researchers within mathematics education who offer a social critique of mathematics teaching.[74] For them, the focus seems to be almost exclusively on instructional approaches and institutional biases. There are thus two levels of critique that must be considered: analyzing the biases implicit in the subject matter and investigating the more hidden prejudices that are enacted as the subject matter is re-presented to students.

Re-Membering Dewey

John Dewey, who was a contemporary of Franklin Bobbitt, articulated a conception of curriculum that was much different from the prevailing modernist perspectives. Briefly, he argued that curriculum had to do with the dynamic and complex relationships among children, teachers, and culture. He thus sought to erase the rigid boundaries that had been drawn between the learner and the curriculum, contending that the two were not distinct but intertwined. For him, the same fluid and co-emergent relationship existed between knowledge—a source of curriculum—and society.

Unfortunately, through the contributions of Bobbitt and others (among whom Ralph Tyler figures prominently), Dewey's work was pushed to the side, and it is only recently that this trend has been seriously challenged. In the mid-seventies, William Pinar[75] introduced the term "reconceptualist" to the field of curriculum inquiry in an edited volume of essays from theorists who had begun to question the field's underlying instrumental (and, arguably, mathematical) rationality. Arriving from backgrounds in literary, existential, critical, feminist, and phenomenological theory, these scholars called

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71 Walkerdine, *The Mastery of Reason: Cognitive Development and the Production of Rationality*, 186.

72 Bookchin, *The Philosophy of Social Ecology: Essays on Dialectical Naturalism.*

73 Orr, *Ecological Literacy: Education and the Transition to a Postmodern World.*

74 Some examples are in order here. With regard to "the gender issue," Elizabeth Fennema (see Elizabeth Fennema and M. J. Ayer, eds., *Women and Education: Equity or Equality?* (Berkeley, CA: McCutchan, 1984).) has written extensively and insightfully on societal traditions and educational practices that serve to militate against the success of females in mathematics schooling. Jean Anyon ("Social Class and the Hidden Curriculum of Work," in the *Journal of Education* 162 (1980): 67-92) investigated different approaches to mathematics instruction and has provided us with a penetrating analysis of the relationships between social class and educational experience.

75 William F. Pinar, ed., *Curriculum Theorizing: The Reconceptualists* (Berkeley, CA: McCutchan, 1975).

for a greater awareness of the ecologies of our existence, the agency of the learner, the interconnections and interdependencies of knowledge areas, and the value of diversity, thus opening the door to a new form of curriculum study. They did so not merely by offering an effective critique of conventional practices, but by reminding us that education is never merely concerned with questions of knowledge.

Put differently, curriculum reconceptualists called into question the modern priority of epistemology over ontology[76]—an emphasis which perhaps arises from the Cartesian assumption that the mind has an existence that is independent of experience. If one accepts the notion that there is a stable Self which precedes learning and which is able to maintain its integrity through learning, then the practices of pre-selecting what is valuable to know and pre-determining how it is best learned are entirely unproblematic. But if one rejects this notion—as do the reconceptualists (along with thinkers such as Merleau-Ponty, Bruner, and Varela et al.)—the alternative is that we are in fact the product of our experiences and, because we are social beings, our minds and our identities emerge and evolve relationally. The issues of learning, teaching, and curriculum, then, are fundamentally ontological, regardless of one's opinions on the moral status of the subject matter at hand.

Further to the issue of knowing, curriculum reconceptualists also challenge the very possibility of pre-determining what is to be learned. A complex and poorly understood phenomenon, human learning has proven itself to be tremendously adept (but wildly unpredictable) at adapting to the contingencies of existence: one never knows exactly what one will learn. Curriculum makers, it seems, have disregarded this commonplace understanding, electing to work from the maxim that what is to be learned can be controlled through careful articulation. The reconceptualist movement, in contrast, might be understood as a return to an acknowledgment of the ambiguities and uncertainties of life. Curriculum is thus not conceived of in terms of distinct (but coherent) knowledge bits, but as having to do with the existential quality of life in schools.

Not surprisingly, then, part of the reconceptualist project has involved an effort to "free" the notion of curriculum from its modern divisive, prescriptive, and instrumentalist frame. To this end, Pinar and Madeleine Grumet[77] have reminded us of the verb *currere* from which "curriculum" is derived. *Currere* refers to "the running of the course" rather than the "course to be run, or the artifacts employed in the running of the course,"[78] and Pinar and Grumet use the term to re-focus our attention away from the impersonal goals of conventional curriculum projects and onto the meaning-making process of moving though the melée of present events. In rendering experience meaningful, one recovers and recreates one's history and simultaneously creates new possibilities for one's future. Such sense-making is understood to be both enabled and constrained by language, and, as such, fundamentally social and relational. In brief, "curriculum"—far from popular conceptions—is conceived as the interpretation of lived experience, and is thus valued for its transformative rather than its transmissive potential. Implicit in the notion of *currere*

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[76] The terms "epistemology" and "ontology" have been subject to a wide range of interpretations. In particular, the term "ontology" is often associated with a conception of the world as pre-existent and accessible to the senses. I do not wish to conflate ontology with metaphysical conceptions of reality. The term is used in this context to call attention to issues of identity and existence—*being*—which tend to be disregarded in discussions of (particularly mathematics) curriculum.

[77] William F. Pinar and Madeleine R. Grumet, *Toward a Poor Curriculum* (Dubuque, IA: Kendall/Hunt Publishing Company, 1976).

[78] Ibid., 18.

are an acknowledgment of the relational basis of our knowing (and being) and a recognition of the happenstantial, constantly negotiated nature of our existence.

It is important to emphasize that Pinar and Grumet are not recommending that standardized curricula be abandoned and new documents reflecting the fluid natures of cultural knowledge and personal identity be developed. On the contrary, such action would miss the point by supporting, rather than challenging, the belief that learning outcomes can be managed. Rather, they are inviting us to *think differently*—not just about the topics of study, but *within* the context of study.

Mathematical Currere

How then to move from instrumental formulations and interpretations of mathematics curriculum? A possible tack may lie in the suggestion that the more fluid form of *currere* points away from the *prescriptive* (retentive) efforts of conventional perspectives on curriculum and toward a more *proscriptive* (attentive) orientation. Following Varela,[79] the difference between *prescription* and *proscription* is essentially the difference between "what is not allowed is forbidden" and "what is not forbidden is allowed"—a shift which might enable us to overcome our modern desire to fix what is learned by mandating outcomes.

Common to both *prescriptive* and *proscriptive* is the root *script*—to write or to draw. *Prescription* is a writing that occurs in advance, a charting of a particular path; *proscription* is a scribing not of route but of boundaries, and it occurs in the immediacy of moving through the bounded territory. One's focus is thus set not on the path (because the course has not been pre-determined) but on negotiating a path—on *currere*, running. The contents of a mandated program of studies might thus be interpreted as outlining areas for exploration, rather than as specifying where each step will land. The curriculum becomes the path that was taken, in all its experiential richness, and can hence be only discussed in retrospect.

Such an interpretation is in greater harmony with the enactivist premise that our identities are established in the dialectical play of biology and human culture. Translating this notion into the realm of mathematics learning gives rise to the suggestions that, first and foremost, learners must come to understand that mathematics is *about* them—where the word "about," in a deliciously ambiguous play, invites at least a four-fold interpretation. "Mathematics is about oneself" simultaneously suggests senses of being surrounded ("round about"), of being the object of focus ("about this idea"), and of being active ("about one's business"). "About" also points to the fundamental interpretability of things. "Tell me about . . ." is an invitation to explain and to re-think. For the current purposes, then, the term is a reminder of the ecological, enactive, and hermeneutic foundations of this project.

How might one foster a sense of *about*-ness in a mathematics classroom? I think that Heidegger[80] provides us with a possibility as he draws a distinction between *mathematics* and the *mathematical*. As I understand him, *mathematics* is that more-or-less static, widely-accepted assemblage of concepts and activities that have emerged through centuries of inquiry. Different sorts of *mathematics* can and do arise, as

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[79] Francisco Varela, "Laying Down a Path in Walking," in *GAIA, A Way of Knowing*, ed. William Irwin Thompson (Hudson, NY: Lindisfarne, 1987).
[80] Heidegger, *Basic Writings*.

illustrated in the preceding section, depending on the era, the culture, and the needs or events that present themselves.[81]

The *mathematical*, in contrast, is that orientation to inquiry which has allowed our *mathematics* to emerge. It involves a noticing of sameness, pattern, and regularity amid one's explorations. It involves comparing, ordering, creating, and naming. It is, true to its etymology and to Pythagoras' definition, about learning. And it is thus that, as some historians contend, *mathematics* (as an independent discipline) may be only a recent phenomenon, whereas there are traces of the *mathematical* in the earliest of human records.[82]

In our own (formal and literate) traditions, we have tended to focus only on the endpoint of mathematical inquiry (i.e., where the *mathematical* becomes *mathematics*)—that is, on the logical situating of a "truth" amid an already established set of propositions. These propositions can be, and often are, modified, as might be the rules of logic that govern the *mathematical* play. But the issue is neither the content of a specific system of mathematics nor the mysterious qualities that enable mathematical intuitions. It is, rather, the particular structure of a mathematical argument—and it is this structure that, in Western history at least, has remained more-or-less constant. As such "situating" or "proving" has become the hallmark of acceptable mathematics, the underlying mode of deductive reasoning has been made equivalent to *mathematical* thought.

As such, with regard to our modern heritage, particular aspects of the *mathematical* have been privileged over the past few centuries, including abstraction, formalization, rigor, and generalization—and perhaps the quality that most distinguishes modern mathematics from the mathematical activity of our ancestors is its current level of formality (i.e., the assigning of form through some manner of representation). The resulting *mathematics* has risen in status alongside the "power" it has offered in the Empiricist project in controlling our world through a knowledge of it. But, as has been noted, this mathematics is not without its shortcomings. We hardly need to know it better or to apply it more effectively. Rather, we must begin to understand that, contra Descartes' belief, mathematical thought simply does not work everywhere. Perhaps, then, we need to seek or to reclaim a different orientation to the mathematical—one that makes it part of the way we participate in the universe.

On this point, curriculum makers (reflecting the modern tendency) have concentrated almost exclusively on the analytic qualities of the mathematical: abstraction, classification, rigor, formalization, "power." While important, these qualities have eclipsed others, such as elegance, patterning, rhythm, and contextualization—elements which clothed the "mathematics of oral traditions." Further, the conversational or

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[81] I might add that one need not go far to observe the emergence of mathematics that diverges significantly from established systems. Indeed, it may not be unreasonable to suggest that, in spite our efforts to prescribe understandings, the mathematics of any given classroom setting will likely diverge in some way from the expected norms. As teacher and learners interact, as they establish their own body of knowledge (i.e., patterns of acting that give rise to a particular collective identity or character), their mathematics will drift from "standard knowledge," even while being framed by that knowledge. It is thus that an investigation into combining fractions might be deflected into a study of partition theory, or a lesson in ratios might shift into a discussion of various forms of reasoning. While the understandings may lack the rigor to qualify as formal mathematics, such contextually specific knowledge is certainly appropriately labeled "mathematical."

[82] I am referring here to cultures that have developed some sort of writing or symbolization processes—a reference that is implicit in the phrase "human records." As argued in the preceding section, the technology of symbolization is likely necessary for the conception of the mathematical as articulated here.

dialogical nature of mathematical inquiry—that is, the aspect of learning that involves an active questioning of the world—has given way to a perception of mathematics learning as solitary and monological. It is here, it seems, that we might most effectively direct our efforts in moving from an orientation to curriculum as a plan to an understanding of curriculum as *currere*. Stated otherwise, the particular concepts (i.e., the *mathematics*) that we select to study, while important, are not as central a concern as the manner in which we choose to portray the *mathematical*.

By drawing this distinction between mathematics and the mathematical, then, a space is opened to develop a critical understanding of some of the defining qualities of our culture and, hence, of our selves. It further serves to emphasize the temporal and contextual nature of mathematics. Finally, it compels us to connect ourselves to the mathematics, for it is we who think this way, and it is this that affects the way we think.

"Subject Matter"

In the first chapter, I described Gadamer's notion of "subject matter" as that which "conducts" or animates a conversation. It is of vital interest to all participants, for the raison d'être of the conversation is to come to some consensus—a common sense—on the issue.

The term "subject matter" also figures prominently in the pages of the curriculum guides and standardized textbooks that line teachers' bookshelves. Far from the subject matter of the conversation, to which the participants subject themselves without reserve, classroom learners are often unwillingly subjected to the subject matter of school mathematics. In another example of our language forgetting itself, the *subject* of mathematics—whether one believes it to be discovered or created[83]—is transformed into an *object* which is then thrown at learners. The hope is that they catch it.

The calls for *currere* over curriculum, proscription over prescription, and the mathematical over mathematics thus amount to a call to regard the "content" as potential "subject matter" (in the Gadamerian sense). It is a call to consider mathematics more as an approach to knowing than as an established body of knowledge. As Max van Manen reminds us:

> To know a subject does not only mean to know it well and to know it seriously in the fundamental questions it poses. To know a subject also means to hold this knowledge in a way which shows that it is loved and respected for what it is and the way it lets itself be known. We learn *about* the subjects contained in a school curriculum. It is also true that the subjects *let* us learn something about them. It is in this letting us know that subject matter becomes a true subject: a subject which makes relationships possible. Our responsiveness, our "listening" to the subject, constitutes the very essence of the relationship between student and subject matter.[84]

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[83] Ernest, in *The Philosophy of Mathematics Education*, develops a theory to "explain" how subjective mathematical knowledge becomes "objective." His explanation is relevant to the current discussion—for it reveals how tightly we wish to cling to our modern ideal of objectivity. A consequence of Ernest's desire to retain a rigid subject-object dichotomy is that he is compelled to articulate a perspective on school mathematics that varies little from the structures he initially sets out to critique. It continues to focus on content- and goals-oriented curriculum, standardized testing, streaming, etc.

[84] Max van Manen, *The Tone of Teaching* (Richmond Hill, ON: Scholastic-TAB, 1986), 45. (original emphasis)

The teacher is called to *listen*, and the focus of that listening is an emergent knowledge—the individual's conceptualizations and the collective realizations—by which the mathematics is not (and indeed cannot be) considered apart from the mathematizers. The mathematics, like the curriculum, is in the realm of the sonorous rather than the visual, thus merging with and emerging from experience. The notion of curriculum, then, involves more than the study of particular ideas, it becomes an integral part of the constantly emerging text of our existence as enacted in the relationships of the classroom. Issues of knowledge and understanding are thus woven into and cannot be considered apart from the notion of identity.

Of course, such "subject matter" can exist only in the conversation. For the teacher, then, the critical issue preceding his entrance into the classroom is how he might transform a curriculum objective into a potential "topic of conversation." The task is not an easy one, for militating against the possibility of falling into a conversation, the mathematics classroom is founded on a particular disdain for that sort of happenstantial interaction. The mere practice of assembling at a specific time and in a specific place thirty persons with diverse interests and concerns might be considered a daunting barrier to any sort of sincere engagement.

As such, simple attempts to rethink the subject matter, although necessary, are patently inadequate. A range of other issues must also be addressed—not the least of which are the natures of the relationships between the teacher and the learners and among learners. These topics are discussed in the next chapter.

For the moment, however, I would like to more specifically address the issue of "planning a lesson." I do so through a specific example drawn from my research. That presentation is preceded with an acknowledgment of its necessary "flatness" and a reminder of my conviction that the content is only one aspect of the classroom's rich texture of relationships. As such, a "learning objective" can only become subject matter through the sensitivity of a *listening* teacher—one who is attuned to the larger context and who is able to bring learners to the ontological significance of the *mathematical*.

Section C

Mathematics Curriculum Anticipating

> [We] need to see . . . that curious consequences flow from planning when this planned instructional program becomes too fixed, too inflexible, too prescriptive for life with children. For one thing, inflexible planning may freeze the body of knowledge that is otherwise dynamic, vibrant, and alive.
> —Max van Manen[85]

One aspect of my research into understanding teaching as a listening endeavor has been to work alongside a grade eight mathematics teacher. Together, we explored many issues relevant to the teaching of mathematics, but the most engaging topic was certainly the nature of mathematical knowledge.

Typical of many teachers (including myself until only very recently), Donna had not thought to look beyond the curriculum guides or her own schooling for an understanding of mathematics. Not surprisingly, then, when I first visited her classroom, hers was a "textbook" approach to planning and teaching.

The question, What is mathematics?, however, served to uncover and disrupt much of what she and I had long taken for granted. Our discussions and conversations on the topic ranged broadly, but there were two clear foci: The first was a head-on struggle with the orienting question (and many of our thoughts on the issue have served to give shape to the preceding sections of this chapter); the second was a "mucking about" with possibilities for teaching. Recognizing the cultural, governmental, and institutional constraints that had to be negotiated, what sorts of lessons could one prepare that would be informed by recent philosophical developments and yet remain viable in the given setting? And how does one go about planning for such lessons?

Perhaps very fortunately, Donna had taken a leave of absence by the time we began our focused exploration of this issue. There is no doubt that the separation from the immediate responsibilities and the constraints—the "realities"—of the classroom allowed us to be somewhat more free-ranging in our thinking.

This separation, however, also served to limit our discussions and conversations as it demanded that knowledge be considered apart from the particular knowers. We thus found ourselves continuously resisting the tendency to speak of the ubiquitous and generic (and therefore non-existent) "student." Nevertheless, we were firm in our conviction that, even if one could not pre-determine the path that would ultimately be taken, the teacher could not abdicate her responsibility to point to particular aspects of the world. Positioned between the learner and society, she has a primary responsibility in shaping the occasions for learning—a responsibility (demanding a response-ability) that moves in both directions. Put differently, the task of exploring teaching ideas—whether in the learners' presence or in their absence—has the effect of broadening the possibilities for teaching responsively. In particular, with reference to the notion of *teaching as listening*, such planning tasks compel the teacher to consider not just the classroom activities, but a range of students responses. The teacher is thus loosening up or

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85 Max van Manen, *The Tact of Teaching* (Toronto, ON: The Althouse Press, 1990), 103.

expanding the prejudices that constrain her listening. The possibilities for her attentiveness and responsiveness are correspondingly enabled.

The goal that Donna and I set for ourselves was thus to identify and develop some teaching ideas for a given unit of study—in this case, introductory geometry. Our hope was that these ideas, when appropriately revised for introduction to a particular group of learners, would support a model of classroom mathematics that foregrounded the *mathematical* while avoiding the modern tendency to sever knowledge from action, interaction, and context. It was thus necessary to consider how one might approach a collection of concepts in a way that would do more than simply sidestep the problems of fragmentation and absolutism; we wanted to create the sorts of conditions that would challenge and disarm the expectations of "math" and "math class" that would likely precede a group of school-weary adolescents into the classroom.

The Topic of Study and Teacher Knowledge

Donna chose the topic of geometry, prompted in part by her feelings of dissatisfaction with previous teaching efforts in that unit. She was also concerned that her own knowledge of the area did not extend far enough beyond the contents of the approved textbook.

Although it did not figure into her selection, the topic of introductory geometry proved to be an ideal one, in part because the strand of geometry runs clearly and distinctly through the history of mathematics. It was a prominent area of inquiry for most, if not all, ancient civilizations. It also served as the centrepiece for the mathematics of ancient Greece and, as such, heralded the rise of formalist mathematics. Indeed, it was this *geometria* on which Descartes founded his rationalist program.

And it was the introduction of non-Euclidian geometries that first hinted that mathematics need not be constrained by intuition, thus prompting the hyper-formalist project. More recently, non-linear dynamics and chaos theory—central movements affecting the emerging shape of our post-formalist conception of mathematics—rely heavily on "fractal geometry," a geometry that "mirrors a universe that is rough, not rounded, scabrous, not smooth,"[86] to break from the modernist ways of thinking.

More romantically, in its very title, *geometry* still echoes the name of Gaia, calling us back to the earth, to the source of our being, to the genesis of our knowledge. Geometry, in its remotest origins, was likely an exemplar of ecological thought. Rising out of the efforts of our ancestors to, quite literally, measure the earth, one can envision those earliest mathematicians on their hands and knees—ears pressed to the ground, as it were—coaxing secrets from the living Earth.

Geometry has also figured prominently in various incarnations of school mathematics, and the topic has actually been identified as the "problem child of the mathematics curriculum."[87] Much of the debate surrounding geometry arises from our general inability to resolve the issues of, first, what geometry is, and second, why we should study it. It is thus that, as curriculum changes have been proposed and implemented, the face of geometry has altered dramatically and its revised images have

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[86] Gleick, *Chaos: Making a New Science*, 94.

[87] Marjorie Senechal, "Shape," in *On the Shoulders of Giants: New Approaches to Numeracy*, ed. Lynn Arthur Steen (Washington, DC: National Academy Press, 1990), 175.

reflected the shifts in our thinking about mathematics and education. For example, the changes that occurred in formal curriculum during the early 1960s, popularly known as the "new math," could be accurately illustrated with a brief description of the increased emphasis on formal geometric proof. More recently, some mathematics educators have recommended that learners be introduced to a few ideas from fractal geometry, a change that could have profound implications not just for mathematics teaching, but for the prevailing conception of mathematics.

One of the issues that inevitably emerges in this sort of analysis, however, involves the level of subject area "mastery" that the teacher must have. The issue is not a simple one, for as Alba Thompson[88] explains, one's approach to mathematics teaching arises from a host of influences, including one's history with teachers, teaching, learners, and learning. The extent of one's mathematical knowledge, while an issue, can be eclipsed by one's perspectives on knowledge or made irrelevant in the face of institutional constraints. A broad knowledge base hardly ensures a pedagogy that is not prescriptive or mechanistic in nature.

Nevertheless, a teacher's lack (or perceived lack) of knowledge can have a severely constraining effect on what is taught. Donna's situation, for example, was not an unusual one. Concerned with the limits of her own background in geometry, she had serious reservations about straying far afield from the paths she had taken, first in being taught the unit and later in teaching it. In particular, in wanting to become more attentive to student action and more open to the mathematical possibilities that present themselves in the course of interacting with learners, Donna wondered if her own limited background would enable her to foster the sorts of conditions that would allow for a more engaged exploration, let alone enable her to notice the possibilities as they arose.

In our conversations on the issue, however, we also agreed that "not knowing everything" was a necessary prerequisite to a more improvisational mode of teaching. Put simply, we don't enjoy interacting with know-it-alls, for genuine conversation is founded on a sort of inquisitiveness on the part of all participants. Analogously, if the teacher believes that she has nothing to learn, there is little reason to be attentive. One need be concerned only with how well the outcomes of learning mesh with the projected results.

To this end, Donna proved to be an ideal partner in this aspect of the research. As we investigated various teaching ideas, her enthusiasm for learning and her enjoyment of mathematical inquiry came through in a procession of "I wonder what happens if . . . ," "Oh, that's what that means!," and "Wow! That's interesting," that pepper the transcripts.

Curriculum Anticipating

A central point here is that the exploratory process of "curriculum anticipating" (versus the narrowing process of "curriculum planning") served to familiarize us with some of the possibilities for the teaching of an introductory geometry unit while it presented an opportunity to enact an ecologically-minded model of teaching and learning. Our investigations were motivated by sincere interests and oriented by particular questions. We were not seeking *the* truth, *the* answer, or *the* way; the goal was to understand more deeply an issue that we held between us. As such, the talk ranged widely: we brought in what we knew and did not know of history, philosophy, and mathematics; we compared teaching and learning experiences; and, most importantly, we

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88 Alba G. Thompson, "The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice," in *Educational Studies in Mathematics* 15 (1984): 105-112.

created a forum in which to re-interpret what we already knew. Put more formally, the conditions necessary for a hermeneutic inquiry were established, leading naturally (although still demanding considerable discipline and effort) to back and forth movements between our conceptions and the web from which those conceptions arose and between the particularity of our teaching practice and the social context in which that practice occurs. The effect of this aspect of the research, not surprisingly, was that more questions were raised than were answered. This consequence was not a negative one, for it instilled a thoughtfulness—that is, it prevented a thoughtlessness that springs from certainty—that deepened the interactions rather than disabled them.

An interesting side-note is that, of the many pages of records and field notes that were assembled over our year-long collaboration, it was the interactions on the topic of planning for teaching that were the most difficult to transcribe. They lacked the I-speak-then-you-speak structure of most of our discussions. Instead, they were filled with interruptions, pauses, incomplete thoughts, exclamations, laughter. In short, they were conversations, and they thus resisted the flattening process of transcription. Unlike many of the other recorded interactions, the tone of these conversations was not what-I-think; what-you-think, but more toward what-we-think. The ideas were thus neither hers nor mine, but ours.

In these conversations we also, I would contend, set a foundation for a "community of professionals" of a sort that is conspicuously (and tragically) absent in most "educational" settings. Lacking such a community, a teacher might understandably not be enthusiastic about departing from "what works"—falling back on planning outcomes rather than anticipating possibilities—for they have no interactive model on which to found their teaching. It is doubtful that the same richness of ideas would have come about were Donna and I to have independently undertaken the task of planning. Within the forum we created, we simultaneously anticipated learner actions and enacted the very model of teaching and learning that we hope could later be brought to life in the classroom. In effect, the multi-layered, self-similar, recursive, and negotiatory natures of teaching and learning were revealed in this rather ordinary (as we perceived it at the time) event.

It was for the expressed purpose of anticipating possibilities that we entered into this aspect of the research. Underlying our conversations was the assumption that the teacher has a responsibility to carefully think through what might happen and to make particular decisions as to the possible shapes of lessons. It is thus that, contra the conventional emphasis on planning (which is guided by a desire to control all aspects of the learning setting), I have elected to use the term "anticipating" to draw attention to our hope to prompt rather than to prescribe appropriate action. The shift in orientation has two important effects. First, as already elaborated, it enables one's listening as it widens the range of one's hearing. Second, by creating a sense of direction and a feel for the territory to be covered, it helps to promote a sense of when to let things diverge and when to re-direct action. In effect, they serve to open a space to learn from what the learners are doing and, from there, to select what might be done next. To repeat, such anticipating activities enable the teacher's response-ability while not permitting an abdication of her responsibility.

A brief account of the process of our anticipating will help to set the stage for the "plan" that we eventually prepared. The activity occupied several meetings, arising out of and occurring alongside our ongoing explorations of the nature of mathematical knowledge. After deciding to translate some of the concepts we were investigating into classroom ideas, we began by reporting to one another some of our past teaching and learning experiences. To our initial surprise, there was considerable correspondence

between our histories, and the many "coincidences" quickly became the targets of our analysis as we began to interrogate the taken-for-granted notions that we had brought to our respective professional practices. Key issues that arose in these pre-anticipating moments included the roles of textbooks and resource manuals (which had been central to both our geometry units) and, as topics of relevance and interest arose, the rather daunting question of why we were bothering to teach the subject matter in the first place.

It was at this stage that the ideas of considering the *mathematical* (rather than concentrating solely on the *mathematics*) and of endeavoring to present a subject matter that was *about* (and not apart from) learners came up. Our orientation as we began our anticipating activities might thus be expressed by the question, How might geometry help learners to better understand their worlds? Or, phrased in a way that makes more explicit reference to the teacher, What occasions could be presented that might prompt learners to act so that their mathematical perceptions and patterns of acting—their structures—are broadened and enabled?

We set off by gathering ideas through exploring curriculum support documents, inviting others to participate, and jotting down things that came to mind. We soon had a collection of possibilities to begin "playing around" with. At one point, for example, a pile of rulers, compasses, protractors, string, roadmaps, and paper polygons—among other items—was stacked between us.[89] There we tested out ideas, guessed what sorts of things might happen, and tried to articulate the necessary pre-conditions for such events. We then selected activities and made a tentative outline (see Table 2) of the teaching ideas along with the content those ideas might support. Having chosen the sorts of spaces that we thought might foster the development of rich repertoires of experience and common action among learners, we then sought to draw out of our own "playing" the elements of the mathematical that seemed to be announced by these activities.

Throughout these explorations, we endeavored to leave as much space for movement as possible, electing to give only brief orienting descriptions rather than detailed mappings. The extent of our investigations is thus not at all represented in the conciseness of the "unit plan." Simply put, we worried that greater detail might prompt us to re-enact the scripts of our explorative activities—an effort that would, in effect, narrow the prejudices we had struggled to broaden.

Further, we felt the sparse detail would compel us to re-anticipate possibilities as the event of teaching approached. This must be a continuing act, preceding, accompanying, and following the introduction of any learning activity. This ongoing attentiveness is essentially a *listening to* (that is, a participation in and an interrogation of) the mathematics of the setting.

Necessary but not sufficient

Curriculum anticipating is thus a necessary—but far from sufficient—condition to enacting a listening orientation to teaching. The teacher is able to consider how particular ideas are connected to other concepts, to bodily experience, to the community of knowers, to the relational world in which it is constituted—in short, to create a basis for an ecologically sound mode of teaching.

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[89] I might note here that the transcriptions of this meeting are essentially incomprehensible—partly because the objects of reference are unavailable to the reader, but mostly because much of our interaction was, simply put, coupled. Persons outside the interactive unity (like me, a year after the fact) simply cannot participate in the enacted meanings.

But such preparation inevitably falls far short of the ideals set out in the preceding parts of this chapter. The teaching ideas remain largely confined to the mathematics classroom, physically separated both from the "real" world and from other disciplines. Its possibilities for promoting understandings of the pervasive shaping force of mathematics within our culture is thus similarly constrained. Also at risk are the desired ends: developing a critical awareness of the consequences of the application of one's mathematics and promoting a certain wisdom toward such application.

On the other side of the issue, however, I'm not sure we should even attempt to write these things into a proposal for teaching, for the mathematics can only present a context for the possible exploration of the mathematical (and other epistemological and ontological concerns). It is only in the actual teaching—a phenomenon that remains to be investigated—that the subject matter actually comes into existence.

Some Anticipating

A summary of some of the ideas that Donna and I explored is presented in Table 2, below. Given the preceding discussion, I am not going to elaborate on the chart's contents other than to focus the reader's attention onto the third column. On the surface, the activities that are described in the first column (and the corresponding formal curriculum objectives that might be addressed during these activities, as presented in the second column) would appear familiar and unremarkable to an experienced mathematics educator, although they may mark a dramatic departure from a textbook-based program. (This observation is an important one: to recast mathematics teaching in an enactivist frame does not demand another round of curriculum revision.)

But it is the emphasis on the mathematical, as outlined in the third column, that signals the difference between this and more traditional program emphases, for the focus in teaching is shifted from "this is what you must know" (i.e., column two) to "this is what it means to think mathematically." As such, mandated objectives become incidental and serve merely to proscribe the lesson setting.

Table 2. An Overview of Some Curriculum Anticipating

Activity	Possible Content	Rationale/Underlying Concepts
1. **Bag of Shapes** Students are given a bag filled with various polygons. They are asked to devise various means of classification.	• various means of classification; • reasons for classification; • development of various skills (angle and linear measurement, property noticing); • (possibly constructing and using a stretch protractor to compare and measure angles).	Mathematics involves • *abstraction* (i.e., extraction, reduction, and simplification—focusing on a particular trait to the exclusion of others); • *classification*; • *naming* (i.e., we classify to make things simpler. The precise classification scheme is often a matter of convenience. Conversely, technical names may encode the conceptual framework in which we organize things.);
2. **Triangles** The triangles are separated out of the *Bag of Shapes* for further examination.	• property noticing: a) properties of all triangles (e.g., sum of angles, etc.), b) classes of triangles (e.g., classifying by sides, by angles, by symmetry, etc.); • skills: - measuring angles and sides; • constructing (other triangles are constructed looking for more properties. May be a nice segue to quad's—two congruent tri's make a parallelogram).	• *generalization* (i.e., we abstract in order to generalize and predict. Certain properties are common to all Euclidian triangles, other properties can be use to distinguish among triangles.); • *standardization* (i.e., the importance of establishing a set of common terms to refer to the units, properties, or categories that are developed);
3. **Quadrilaterals** As with triangles, the focus shifts to examining quadrilaterals.	• property noticing: a) properties of all quad's (e.g., sum of angles, etc.), b) classes of quad's (e.g., classifying by sides, angles, symmetry, diagonals, etc.); • skills: - measuring of sides & angles; • construction (other quad's are constructed looking for more properties. May want to return to idea that a quad is two triangles, because subsequent polygons with n sides can be constructed using $n - 2$ triangles.).	• *symbolization* (i.e., our ability to express our ideas in symbols allows us to operate on those representations, enabling us to move to other levels of abstraction); • *power* (i.e., if our abstracting and generalizing were appropriate, we can begin to make fairly powerful predictions about other related topics/shapes/concepts); • *elegance* (i.e., some ideas are simple, yet powerful. Mathematicians tend to prefer these ideas.);
4. **Polygons** The exploration of triangles and quadrilaterals is broadened to include other polygons.	• property noticing; • constructions with ruler, protractor, and compass.	• *aesthetics* (i.e., mathematics has long been associated with beauty—principles and products alike).

Table 2. An Overview of Some Curriculum Anticipating (continued)

5. Roadmaps Start with identifying triangles, quadrilaterals and other polygons on the map. Then move more specifically to angles formed by turning corners (turning left versus turning right [supplementary], the sum of the angles when turning around and going the other way [supplementary again], the sum when turning all the way around, the sum when turning twice to change directions, etc. Then move to parallel lines and transversals (which can be seen to be producing the polygons under study).	• further property noticing; • using grids (first quadrant, where street number gives *x* coordinate and avenue gives *y* coordinate); • applying knowledge of polygons by identifying them on the map (IDEA: If you start from some point, go somewhere, and come back on a different route, you've traced out a polygon: a CLOSED figure with STRAIGHT sides); • supplementary and complementary angles; • F, C, Z, and X angles—and maybe others—all as useful ideas for studying shapes, etc. (i.e., means and not ends); • using angle relationships to predict, calculate, and justify other angles; • "least number of angles" questions (i.e., given a complex figure, what is the minimum number of angle measurements so that all the angles can be determined?); • naming angles—the three ways (interior, point, 3-point)—when and why would each method be used?	Math principles are extracted from our experiences. They are useful not only because we can use them to generalize about experiences we've had, but because we can use them to predict future actions (What angle will we have to turn?) **and** to structure or arrange our world (as in the way we set out our cities). Many mathematical ideas are simply conventions that people have agreed on. These include the terms used to refer to pairs of angles, the way we name angles, etc.
6. Complicated Questions Devise hard questions. For example: First on the roadmap and then on a separate piece of paper where you draw the lines, develop questions where certain measurements (angles and otherwise) are missing. Then set questions for classmates such as: • What's the least number of points needed to name all the angles? • What are the missing measurements? • If this angle changed, what would happen to that one?	• problem solving; • applying understandings and skills.	Much (most?) of mathematical inquiry involves the identification of problems and attempting to solve them. Through this process, new ideas are developed, which are used to work on even harder problems. Another essential element of mathematical inquiry is *justification.* While not asking for formal proofs, it's important to foster a sense of the sequence of reasoning involved here.

Further Comments

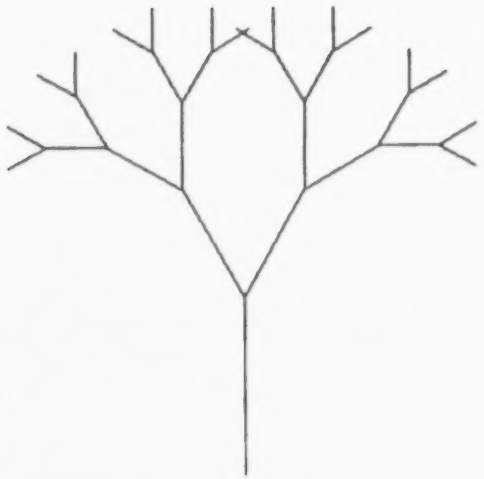
Some additional notes regarding the chart's contents are in order. Most importantly, they are not intended to represent a linear progression, nor to prescribe everything that is to be done. In fact, it is unlikely that all (or even most) of these activities would be undertaken, and even more unlikely that the presented sequence would be maintained. It may well be, for example, that the introductory activities serve as a basis to address all of the issues above; alternatively, some entirely unanticipated event may render these ideas unnecessary. There is no predicting what ideas will come up, what interests will emerge, what insights will arise. This "plan" is thus best thought of as a series of possible prompts or nudges to encourage movement through a mathematical space. It is not a scheme to be implemented, but a series of possible entry points for teaching action. It is, then, merely a starting place for a continuous process of anticipating; it is more along the lines of a strategy for an as-yet unplayed game than an algorithm for reaching a particular destination. It is a way of stepping into the current of a curriculum.

The status of the third column is somewhat ambiguous. By including it, I do not mean to suggest that these things should be taught directly, nor even that the teacher should try to steer the activities in a direction that would "make them fit." Rather, these ideas are intended as a persistent reminder that the mathematics content, while interesting, should not be erected as the end point to learning. In simple terms, if all we learn about angles is how to identify them, we miss the whole point.

In this conception, this unit outline merely marks some places where learning might happen.

Chapter 3

STOOD ON ONE'S EAR
The Educational Endeavor



[A] deep hypocrisy . . . runs through our lamentations about education. The illiteracy of the young turns out to be our own reflected back to us with embarrassing force. We honor ambition, we reward greed, we celebrate materialism, we worship acquisitiveness, we cherish success, and we commercialize the classroom—and then we bark at the young about the gentle arts of the spirit. We recommend history to the kids but rarely consult it ourselves. We make a fuss about ethics but are satisfied to see it taught as an "add-on," as in "ethics in medicine" or "ethics in business"—as if Sunday morning in church could compensate for uninterrupted sinning Monday to Saturday.

The children are onto this game.

—Benjamin R. Barber[1]

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[1] Benjamin R. Barber, "America Skips School," in *Harper's* 287 (November 1993): 39-46. 42.

Section A

Culture Making: The Place of Education

> Teach your children what we have taught our children—that the earth is our mother . . . Man does not weave the web of life; he is merely a strand in it. Whatever he does to the web, he does to himself.
> — Chief Seattle[2]

If the space of curriculum is between our collective knowledge and the knowledge of the learner, then education is situated between the society and the child, between the actual and the possible, between certainty and chaos, between past and future.

Education exists and consists in such relational spaces. They are not difficult to identify; as teachers and learners, we continuously negotiate them. But it is quite another matter to point out the boundaries that mark the *place* of education within these spaces.

Such is my present task. I begin with a brief tour through some prominent orientations (political, theoretical, and personal) to education and to schooling. Following the pattern already established, this exploration involves a critical investigation of a range of modern conceptions, and is followed by an examination of an alternative that is derived from pragmatist, ecological, enactivist, and hermeneutic notions.

But first, an elaboration. In the previous chapter, I used the word "pointing" to describe the curriculum-maker's task of orienting the learner's attention toward particular aspects of our culture. In this discussion of education, as I move closer to the vibrancy of an actual classroom, I am compelled to replace the passive detachment of *pointing* with the active engagement of *interpreting*, for in standing between, in selecting *this* and not *that*, the educator does more than merely direct. She inserts herself into the events—the places—of education.

Places

What is it that makes a place a place?

The answer is simple if one chooses to define "place" formally. It is a location in space, a fixed point (or collection of points) in a given domain.

This very modern definition reveals the influence of one of Descartes' major contributions to contemporary thought and formal mathematics: the Cartesian coordinate system, by which each point in a space is defined in terms of its location relative to an arbitrarily positioned set of axes. This system of *placing* places cuts up a territory with complete disregard for the surface features, the events, and the relationships that are associated with it. Room numbers, city addresses, roadmaps, and schedules are often presented in Cartesian or quasi-Cartesian forms.

But while these coordinates might help us to locate places quickly and accurately, they reveal nothing about the places themselves. Place is more than mere location; in fact,

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[2] Cited in S. van Matre and B. Weiler, *The Earth Speaks* (Warrenville, IL: The Institute for Earth Education, 1983).

such locating usually occurs only after a place has become a place. Only then do are we concerned with exactly where it is. So what is it that brings a place into being?

Again, we tend to have ready answers. A place, for example, may be defined according to its topographic features: some places include "the river valley" and "the education building." Or it may be identified by the sorts of activities that it sponsors (or for which it was intended), or by the sorts of people who gather there.

However, from an enactivist position, each of these means of characterizing a place is inadequate for the purpose of understanding what a place might be. The placeness of a place is never merely a singular matter of physical features or social activities. Rather, such qualities are co-implicated: the topography of a place selects and shapes the actions and relationships that occur there; inversely, in *taking* place, these activities and relationships shape their landscape. We and the places we find ourselves co-emerge; we inhabit and en-habit one another.

One etymology of *place* suggests that the word originally meant "the sole of the foot." A *place*, in other words, was that part of the body that touched the earth. In the moment of contact, the ground conforms to the foot just as the bottom of the foot molds itself to the contours of the surface. Occupant and occupied take shape simultaneously.

In other words, we are part of the places we occupy, just as these places are part of us. This point is powerfully illustrated by the way we describe ourselves, for these descriptions invariably make reference to particular places—to nations, to hometowns, to classrooms, to relationships. Moreover, we tend to implicate our own identities in our descriptions of familiar places. (It is thus that we find ourselves feeling violated when a childhood playground is destroyed or a favorite river is polluted.) We do not define places; they do not define us. Rather, in dynamic interplay, we come to form together.

An inquiry into the place of education and of teaching, then, is not merely an exercise in identifying the location assigned by society, for education and teaching are active in shaping their own landscapes. Their places cannot be reduced to statements of purpose, descriptions of role, or delineations of demographics (as many modern efforts would have it). They are, rather, dynamic and autopoietic (that is, self-shaping and self-maintaining) within a cultural setting, just as they are contained and shaped by that setting. As I explore the places of education and teaching, then, I am not attempting to fix them spatially, temporally, or socially. Rather, I am seeking out where they place us as I explore where they are placed and where they take place.

I begin with the premise that our system of education is an integral part of our culture, shaping not just individuals but the collective psyche. As such, it is a profoundly moral endeavor, and so we dare not consider its place with the indifference and detachment of a (Des)cartographer.

(Mathematics) Education—Some Perspectives

To educate is to engage in an intentional activity—it is directed toward some already-anticipated end. This end varies according to one's perspectives on knowledge, one's ideology, one's social group, and a host of other factors ranging from the subtle to the imposing.

Given the multi-textured surface of our own society, it is to be expected that there are profound disagreements among persons regarding the role of education (and,

correspondingly, on such issues as the structure of the schooling system, the approach to instruction, and the nature of the teacher-student relationship). And given the pervasiveness of modern (rationalist and empiricist) orientations to dealing with the world, it is not surprising that the predominant perspectives on education tend to be framed in terms of either "the progress of society" or "the good of the child."

Paul Ernest[3] provides a clear illustration of this point. In a brief survey of various educational ideologies he attempts to identify the perspectives on mathematical knowledge, moral values, society, the child, and other facets of the educational endeavor that, to his analysis, are aligned with each of the prominent orientations. Because his examination serves as a cogent example of not just a modernist analysis (i.e., one that, in the end, is founded on Cartesian assumptions), but of the spectrum of modernist views, I will use it as the springboard for my own interpretations.

Directing his account at the British context (but not irrelevant to our own), Ernest begins by tracing the development of the currently dominant *Industrial Trainer* and *Technological Pragmatist* orientations to education, suggesting that formal schooling has for some time now been controlled by the interests of industry and technology. Emerging from the demands brought on by the Industrial Revolution (and, more recently, the technological revolution of this "computer age"), society had need of disciplined, trained workers who would accept their place in the social hierarchy. Mathematical knowledge, in this radically conservative account, is valued for its utility rather than its truthfulness. For the purposes of educating the populace, then, mathematics (like most other subject areas) is regarded as an unquestioned body of knowledge that is taught because of its usefulness for the technically-literate worker. With a guiding metaphor of mathematics as a set of tools, the focus of instruction is on basic facts and rules. Exposition, repetition, and drill are the foundation of the preferred teaching method.

Associating mathematical competence with the enabling of technology, this orientation to education underlies the oft-heard calls for "back-to-the-basics," supporting the belief that, in times of restraint, "unnecessary" courses such as music and art should be the first ones cut. The technical orientation has also given impetus to the increasingly popular practice of international comparison testing.

Ernest contrasts this perspective with the more liberal attitudes of the *Progressive Educator*[4] who holds a more person-centered view of education. In place of the technologist's authoritarian and practical emphases, progressive educators advocate a more nurturing relationship with the child. Mathematics, as with the other disciplines, is seen as a means of promoting creativity and self-realization. It thus continues to be viewed as neutral knowledge, although its absolute nature is abandoned in favor of a more personal, subjective account of knowing. Privileging the relativistic ideals of independence, personal autonomy, and self-expression, the progressive educator espouses a teaching approach that is based on the facilitation of activity, play, and personal exploration through the provision of "rich" environments.

Finding fault with the ideological, epistemological, and educational implications of both technological and progressivist orientations—primarily on the grounds of their failure to address political and social issues—Ernest proposes his own model of *Public Education*, the central goal of which is the preparation of an informed citizenry capable

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[3] Ernest, *The Philosophy of Mathematics Education*.

[4] It must be emphasized that Ernest's use of "Progressivism," a terms that is often associated with the philosophy of John Dewey, is unrelated to Dewey's philosophy. Ernest's analysis of Progressivism is more toward the work of Rousseau.

of full participation in a democratic society. Not unlike the *Academic Rationalism* of Mortimer Adler[5] and E. D. Hirsch,[6] Ernest's *Public Education* is designed to provide essential knowledge and to promote critical intellectual skills (such as thinking, observing, and communicating). Rejecting the extremes of absolutist and relativist perspectives on knowledge, he calls for an awareness of the social basis of knowledge production. Such an awareness, he argues, will serve to foster critical consciousness and democratic citizenship.

A fourth movement in educational thought, and one which Ernest does not consider, is *Critical Pedagogy*. Its proponents, which include Paulo Freire[7] and Henry Giroux,[8] come from a range of backgrounds, including Marxism, postmodernism, and various feminisms. It aims more toward the rupture of the oppressive tendencies of political regimes (democratic or otherwise) than toward mere conscientious participation. Formal education is thus more tightly linked to activist politics. Giroux explains that

> making the pedagogical more political means inserting schooling directly into the political sphere by arguing that schooling represents both a struggle to define meaning and a struggle over power relations.[9]

As Peter McLaren points out, this perspective is closely aligned with the Foucauldian notion that "all regimes of truth [are] contemporary strategies of containment. The point . . . is to purge what is considered truth of its oppressive and undemocratic elements."[10] Not surprisingly, mathematics, set up as it is as the model of reason and situated as it is at the core of the modern curriculum, is a particular target of these critical theorists.

A Critique

On the surface, it appears that each of these perspectives has its own particular agenda; and, while some are in many ways compatible, proponents of any one are quick to find fault with the others. In this section, I would like to argue that in spite of their differences, these orientations are really much more alike than a cursory glance would reveal.

To begin, in acts of monologic authority, proponents of each of the above perspectives start out by assigning education its place. Boundaries, rationales, and purposes are given in concise terms, and little space for negotiation is permitted. With such clear-cut guidelines, then, all of the orientations are, in one way or another, modern.

The clearest example is the industrial or technological perspective. Guided by the ideals of competitiveness, efficiency, productivity, and progress, the educational system of the Industrial Trainer and the Technological Pragmatist is premised on the separations of knowledge from knower and society from individual. Preference is given to the first terms in these dyads, as the latter are valued only insofar as he or she is an adequately

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[5] See Mortimer J. Adler, *The Paideia Proposal: An Educational Manifesto* (New York: Macmillan, 1982).
[6] See E. D. Hirsch, Jr., *Cultural Literacy: What Every American Needs to Know* (Boston: Houghton Mifflin, 1987).
[7] See Paulo Freire, *Pedagogy of the Oppressed* (New York: Seaview, 1971).
[8] See Henry Giroux, *Teachers as Intellectuals: Toward a Critical Pedagogy of Learning* (Granby, MA: Bergin and Garvey, 1988).
[9] Ibid., 172.
[10] Peter McLaren, in Giroux (Ibid., xx).

functioning component of a machine-like whole. A powerful criticism of this perspective is offered by a number of postmodern and poststructuralist theorists who argue that industrialist ideologies and philosophies are founded on untenable structuralist and positivist assumptions. Cleo Cherryholmes provides an insightful analysis of this orientation, concluding that

> contemporary education is constructed on outmoded and dangerous structural, utilitarian, and instrumental assumptions. They are outmoded because they make rhetorical claims for textbooks, teaching, research, and practice that their logic subverts. They are dangerous because they rhetorically promise foundational, final, and efficient answers about which their logic is silent. They dehumanize by demanding that we adjust to structures imposed upon us while remaining silent about the exercises of power within those structures.[11]

The central themes of this criticism are thus that the interests of the individual are subordinated to those of the institution and the maintenance (or progress) of existing social structures is given precedence over the enabling of learners to rise above their current status.

The Progressive Educator, in contrast, reverses the technocratic priority by placing the "innocent" and ostensibly autonomous child at the centre of the educational endeavor. Far from preparing the learner for the marketplace, the role of the educator is to protect the child from the corrupting influences of the world. As such, the Progressive Educator maintains the same rigid distinction between child and society, although spaces are opened for deeper and richer relationships between teacher and learner and between learner and knowledge. Nevertheless, the progressive education movement appears to have fallen short of its goal to liberate the child. Edwards and Mercer, for example, suggest that even among teachers who describe their practice as progressivist, the instruction is characterized by the teacher "retaining tight control, dominating the agenda and discussion, determining in advance what should happen and what should be discovered," leading to situations where learners are unable to function "outside the precise context and content of what was 'done' in the classroom."[12] In other words, unaware of the modernist ground on which they are founded, and ignoring the social matrix in which they exist, the dynamics and the outcomes of "progressive" classrooms are very similar to those that are more technical in orientation.

Valerie Walkerdine provides another critique, contending that the progressivist movement has failed not because of its emphasis on the individual but because of its focus on "the child." She argues that the progressivist notion of "'the child' is an object of pedagogic and psychological discourses. It does not exist and yet is proved to be real every day in classrooms and laboratories the world over."[13] Put differently, progressivist discourse has failed to recognize that "the child" is not coterminous with actual children.

The most positive quality of the progressivist orientation—an attentiveness to the relationship between teacher and learner—is maintained in the Public Education model, which also attempts to consider more productively the interplay of the individual and the collective. Nevertheless, with such descriptors as "clay molded by environment," it is

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11 Cleo Cherryholmes, *Power and Criticism: Poststructural Investigations in Education* (New York: Teachers College Press, 1988), 186.

12 Derek Edwards and Neil Mercer, *Common Knowledge: The Development of Understanding in the Classroom* (London: Routledge, 1987), 167.

13 Walkerdine, *The Mastery of Reason: Cognitive Development and the Production of Rationality*, 202.

clear that the learner is still considered as a distinct (albeit somewhat shapeless) entity—for the emphasis here is on considering the individual in its environment, not on considering the ecologies of the learner. Further to this point, a criticism that could be leveled against both Ernest's Public Education and Adler and Hirsch's Academic Rationalism is that both models begin with the premise that they represent the most enlightened and culturally-advanced perspective. Like the technocratic argument, this orientation privileges a masculine and Western view of the world.

It is thus that the criteria for determining which aspects of mathematical knowledge are "essential" do not differ greatly between Public Educator and technocratic orientations. Indeed, as Ernest goes into great detail on his model, he finds a means of justifying the rigid and impersonal examination regime of the British educational system—a program that was initially founded on industrialist needs and which has contributed significantly to the maintenance of the social inequities that the Public Educator supposedly seeks to reform. As such, the modern qualities of competition, stratification, and productivity continue to be foregrounded.

The Critical Pedagogy movement might be criticized on the same grounds, for their entire project is founded on the assumption that human relationships are necessarily political and competitive. These relationships are characterized as perpetual power struggles that inevitably result in control and oppression. The modernist influence is clear here, and it is expressed in active (rather than merely descriptive) oppositions: we versus they, emancipation versus subjugation.

But, although the movement has had a valuable and potent effect in alerting us to many of the social inequities of our society, it bears the seeds of its own destruction. Some aspects of Critical Pedagogy might be described as being radically objectivist, for forgotten are the metaphorical origins of the power *structures*, the social *barriers*, and the *regimes* of truth. Once intended as conceptual tools, these constructions have themselves become obstructions in the discourse field, and their reification has forced proponents to assume adversarial postures; no position other than one of strife and struggle is tenable when one refuses to entertain other models or possibilities for human interaction.[14]

In offering this critique of Critical Pedagogy, I do not mean to suggest that the issue of "authority," a central target of the movement, is not an important one. Quite the contrary, it is perhaps *the* issue, for, as Karl Jaspers[15] argues, education might be thought of as a dialectic between authority and freedom. The point being made here is that authority should not be identified with oppressive power or intrusive violence—these place it on a foundation of fear. Rather, authority might better be aligned with reasonableness. Hear Gadamer on this issue:

> This acknowledging authority is always connected with the idea that what authority says is not irrational and arbitrary but can, in principle, be discovered to be true. This is the essence of the authority claimed by the teacher, the superior, the expert. The prejudices they implant are legitimized by the person who presents them.[16]

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[14] Gallagher, in *Hermeneutics and Education*, discusses the link between Critical Pedagogy and critical hermeneutics, pointing out that while the former finds much of its inspiration in the latter, "few attempts have been made to consider or justify critical education theory in terms of critical hermeneutics" (p. 25). As such, the hermeneutical dimension has remained far in the background of the Critical Pedagogy movement.

[15] Karl Jaspers, *Philosophy of Existence* (Philadelphia, PA: University of Pennsylvania Press, 1971).

[16] Gadamer, *Truth and Method*, 220.

A Middle Way

And so, even though these perspectives appear to be diverse and even contradictory, they "share a core set of Cartesian/liberal assumptions."[17] As Bowers and Flinders elaborate:

> That each, while dealing with important issues and appearing to make a convincing case (depending on which part of the Cartesian/liberal paradigm the reader takes for granted), is totally silent about the connection between cultural beliefs and practices and the ecological crisis suggests the basic limitations of the Cartesian/liberal paradigm. Its blindness to the long-term interdependence of cultural patterns and natural environment is reflected in the blind spots of [their positions].[18]

In particular, modernist assumptions or ideals that are common to the perspectives presented include the primacy of the epistemological, the construct of a static Self, the distinction of self from other, a desire for progress, a goal of optimalization, and a belief that competition is a central defining characteristic of existence. In proposing alternatives to these notions, enactivist and ecological theorists also offer a starting place for re-thinking educational philosophy. In this section I attempt to sketch out a basis for these alternatives, setting the stage for an enactivist response to the question, Why educate?

To begin, the issue of the modern tendency to privilege the epistemological over the ontological has already been addressed in the preceding chapters. Briefly, the position taken was that, on issues of knowledge, the question of what we know is not distinct from the question of who we are. Nor can our knowledge be considered as "prior to" or "dependent upon" our identities, for the two are established together. This argument, in effect, is the foundation of the enactivist challenge to the modern construct of a static Self. Not held to be pre-given or fixed, the self, in enactivist terms, is historically and relationally established and is therefore contextually and temporally dependent.

Nor can the self be considered apart from others. In simple terms, the qualities I invoke to give shape to my identity and the stories I construct to give this self an historical coherence are linguistically (and therefore socially) constituted.

The desire for "progress" lies at the core of all modern educational philosophies. Implicit in the ideal of progress is a pre-conceived goal which one must strive to attain. In itself, this notion is not problematic; it is not the goal that presents the immediate difficulty but the inability of the goal-makers to define their goals in terms other than "progress." As David Denton explains:

> [Our] assumption of one overarching purpose, namely, progress has blocked any serious discussion of alternative ends to education. The ends or purposes of education come, without serious reflection, from external units of the society: the state, churches, military, industry, or "the market place."[19]

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[17] C. A. Bowers and David J. Flinders, *Responsive Teaching: An Ecological Approach to Classroom Patterns of Language, Culture, and Thought* (New York: Teachers College Press, 1990), 241.
[18] Ibid.
[19] David E. Denton, *Gaia's Drum: Ancient Voices and Our Children's Future* (Hanover, MA: The Christopher Publishing House, 1991), 25.

When the activity of progress-oriented goal-setting is undertaken from a strictly modernist mind-set, the means of attaining those goals are inevitably articulated in prescriptive terms. Therein lies the problem. The goal of the Industrial Trainer is an efficient and complacent workforce; the means is a technical fact-based use-oriented education. The goal of the Academic Rationalist is a smoothly functioning democratic society; the means is the provision of a homogenous and "truthful" (and therefore uncritical) knowledge base. The goal of the Critical Pedagogue is emancipation; the means is a provocation of the oppressed to some form of violence against their oppressors. In each case, both goal and means can be stated in terms of control or management—modern ideals which bring along the baggage of cost effectiveness and resource-use efficiency—founded on the dream of optimization and on the threats of a more capable competitor or more devious adversary.

This thoroughly Western take on the "progress" of society is consistent with, if not predicated upon, a survival-of-the-fittest mentality that is appropriately associated with the notions of competitiveness and optimization. Challenging this orientation, enactivist theorists Varela at al. offer the metaphor of *bricolage*. In their words, bricolage refers to "the putting together of parts and items in complicated arrays, not because they fulfill some ideal design but simply because they are possible."[20] Phrased differently, a post-Darwinian conception of evolution is not founded on optimization (survival of the *fittest*), but on satisficing (survival of the *fit*). The sole criterion for continued existence is viability—that is, an adequate compatibility with the given context. Replacing the ideal of optimization with the criterion of satisficing, in effect, destroys the basis of our culture's desire for progress, because the goal of that progress—some form of best-ness—were it even within our capacity to anticipate, is (at the most optimistic level) a moving target.

The modernist understanding of "progress" is tightly linked to the privileging of vision over the other senses. If we compare, for example, our *visual space* to our *sonorous space*, we note that we are *placed* differently in these sensory realms. We stand at the edge of our visual space, looking into it, whereas we are placed at the centre of our sonorous space, surrounded and immersed. Visual awareness is unidirectional and forward, and a vision-dominated consciousness is thus occupied with projecting, foresight, speculation, and seeing ahead—the roots of a desire for progress. Audio awareness is omni-directional and centered, and so the listener is more concerned with locale and immediacy.

Hence the listener is not primarily interested in progress but in movement, for movement—and not progress—is a defining characteristic of life. All living things have some capacity for motion. But it is not a movement toward a pre-specified goal (i.e., progress); it is more an intricate dance with other elements of one's ecological sphere in which the players select in one another particular actions or qualities. It is not a machine propelled by competition, but a structure defined by co-emergence.

It is this contrast between modernist and enactivist perspectives which offers the basis for rethinking the place of education in our society, for it offers the hope that education might be something more than a training ground as it is freed from its role of supporting progress. The focus of the educator's efforts are shifted from unattained learning outcomes to current relationships—relationships on levels ranging from the species-biosphere to the learner-classroom. Education is thus not about attaining the best

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20 Varela et al., *The Embodied Mind* (Cambridge: The MIT Press, 1991), 196.

but about "living well in particular places."[21] It moves from an "almost occult yearning of the future"[22] to an embracing of the here and the now. In Robert Young's terms, the goal is "education for life," an education "for reflective change and adaptation of the self, for co-operative change in relationships with others, and holistic and respectful change of the environment we share."[23]

And, as argued in the preceding chapter, it is an education that must include studies of mathematics and of the mathematical.

Mathematics (Education)—Why teach math?

Shifting the discussion to focus more specifically on the reasons for studying mathematics affords me opportunities both to elaborate on these ideas and to revisit the conclusions of the preceding chapter. Just as that analysis of the nature of mathematical knowledge pointed to the necessity of reconsidering what is meant by "subject mater" and "curriculum," this examination of education through the lens of enactivist thought demands a re-evaluation of the practice of situating mathematics at the core of the schooling experience.

Our answers to the simple question, Why teach math?, can be particularly revealing, for we tend to have a range of responses at our fingertips. Embedded as they are in a modernist setting, these responses are often expressed in a tone of certainty that belies their shaky "common sense" groundings. I recently posed the question to a colleague, for example, and her matter-of-fact answer was a terse, "You need it." In spite of my efforts to place a more critical edge on our discussion, her perspective on the issue was neatly summed up in those three words.

My efforts to compile a more comprehensive list of reasons have been frustrated by statements that reflect little more depth. (Indeed, this inability to "justify" what I had done for nearly a decade was the source of considerable unease early in my doctoral program.) In this section I present and critique some of the more popular rationales. To facilitate the discussion, I have arranged them according to the presumed benefactor—one of: the learner, the educational system, or society.

The most commonly cited argument for studying mathematics is that the mere activity is believed to promote reasoning skills, a notion that likely emerges from the alignment of mathematics and reason in our culture. James Fey's remark is typical: "Without question, the most important goal of school mathematics is to develop students' ability to reason intelligently."[24] The difficulty with these sorts of statements is not so much the narrowness of their conception of reason, but the correspondence between the goal as stated and the actual consequences of living through one's mathematics education. Mathematics learning may well support thinking abilities—the research has been contradictory on this issue—but there can be little debate of the point that, at least as often as not, conventional mathematics instruction has quite the contrary effect as students resort to memorization and rote application of largely meaningless procedures.

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[21] Orr, *Ecological Literacy: Education and the Transition to a Postmodern World*, 84.

[22] Berry, *The Unsettling of America: Culture and Agriculture*, 4.

[23] Robert Young, *Critical Theory and Classroom Talk* (Clevedon, GB: Multilingual Matter Ltd., 1992).

[24] James T. Fey, "Quantity," in *On the Shoulders of Giants: New Approaches to Numeracy*, ed. Lynn Arthur Steen (Washington, DC: National Academy Press, 1990), 91.

That is, mathematical concepts tend to be reduced to mere tools rather than means of exercising one's thinking. Alberta Education, for example, offers as one of its three goals and objectives for the Junior High Mathematics Program that students will be able to "use mathematics as a tool in the pursuit of personal goals and aspirations."[25] The modernist ideals of individualism, competitiveness, objectivity, and exploitation permeate the statement.

Mathematics instruction is also defended as a measure of rigor in a learner's program. The number of mathematics courses taken by a student is commonly regarded as an indicator of his or her potential and ability, not in the least because mathematics wears the mask of objectivity and impartiality so effectively. Thus, courses in mathematics have assumed a "weeding out" or "gate-keeping" role. That this role might be antithetical to the notion that the purpose of education is to present opportunities—not to deny them—is often not considered. It is perhaps fortunate, then, that we are continuously confronted with the inappropriateness of this rationale and this practice as a parade of successful public figures flaunt their feelings of mathematical incompetence.

The most pervasive reasons for studying mathematics—and certainly the reasons that are most in harmony with the dominant technical-pragmatic perspectives on education—are those which are couched in terms of societal need, A mathematically-literate citizenry is essential in our technologized world. But here we would do well to question not only the role of school mathematics, but the worthiness of the perceived end. Is it best that we attempt to perpetuate our technological culture in light of the toll it has exacted from the planet? Might it not be more appropriate to shift our educational emphasis to other disciplines and modes of reasoning that can more powerfully connect us to the earth, to our past, to one another?

This discussion would be incomplete without at least a brief glance at a rationale that has become a favorite of government and media alike. Success in the study of mathematics has somehow become closely linked to our nation's economic prosperity. Study after study indicates that Japanese and German students outperform their Canadian counterparts. Leaving aside cultural and methodological issues that would render problematic these studies' findings, and ignoring the Cartesian assumptions that underpin this practice of comparative examination, there seems a need to question how it might be that our economic ills can be in any way attributed to the performance of twelve-year-olds on a standardized test which is of little consequence to those writing. Madeleine Grumet[26] contends that this practice represents an attempt by the governing males to deflect the blame for their near-sighted decisions onto those who will suffer the consequences: the children who fail and the women hired to teach them. David Orr suggests that our misplaced concern over our children's lack of competitiveness in science and mathematics has prevented us from attending to whether or not they "will know how to protect the biological resources upon which any economy ultimately depends."[27]

In sum, then, when analyzing the situation through an ecological or enactivist framework, we are unable to provide a satisfactory defense for the mandatory study of mathematics in its current form. Quite the contrary, it is easier to argue for a relaxation of

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[25] Alberta Education, *Junior High Mathematics: Teacher Resource Manual* (Edmonton, AB: The Crown in Right of Alberta, 1988), 2.

[26] Grumet, *Bitter Milk: Women and Teaching*.

[27] Orr, *Ecological Literacy: Education and the Transition to a Postmodern World*, 83.

the requirements—and this is certainly the position of certain feminist[28] and ecological[29] theorists.

A Possible Rationale

It seems that we are trapped in an untenable position: between seeking to preserve what we hold to be a valuable part of our intellectual heritage and avoiding the perpetuation of a dehumanizing practice.

But what must be borne in mind is that, as elaborated in Chapter 2, with the realization of Descartes' dream of world mathematization, we have become as much the products of mathematics as it is the product of us. We see the world in mathematical ways as aspects of the discipline infuse our language and frame our experiences. To understand the universe in which we find ourselves and in which our selves are established, the study of mathematics, like studies of language and history and art and music, is critical. The suggestion that we disregard a discipline is tantamount to the recommendation that we ignore who we are and who we might be—and what our society is and what it might be. We study mathematics not to master its processes or to possess its objects, but to understand the world into which we are thrown and which we participate in creating.

Further to this line of reasoning, in pointing to the notions that our personal and collective identities are interactively established, that the fundamental unit of survival is not the organism but the "flexible organism-in-its-environment,"[30] and that we belong to our history and not it to us, enactivist and ecological theories have effectively thrust the educational endeavor into the realm of the ethical.[31] As Wendell Berry argues, "under the discipline of unity, knowledge and morality come together. . . . To know anything at all becomes a moral predicament."[32] And in highlighting the notion that even the slightest perturbation can have the most profound effect when processes of repetition and recursion come into play, they have also provided us with the moral imperative to intervene in a particular way.

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[28] E.g., Walkerdine, *The Mastery of Reason: Cognitive Development and the Production of Rationality*; Nancy Shelley, *Mathematics is a language*, presented at the Seventh International Congress for Mathematics Education (Quebec City, PQ, August, 1992).

[29] E.g., Bookchin, *The Philosophy of Social Ecology: Essays on Dialectical Naturalism*; Orr, *Ecological Literacy: Education and the Transition to a Postmodern World*.

[30] Bateson, *Steps to an Ecology of Mind*, 45.

[31] My uses of the terms "moral" and "ethical" here are according to a cooperative rather than a competitive basis for human interaction. As Levin, in *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, elaborates:

> The two different ways of thinking about moral problems are (1) a competitive model, which gives primacy to the individual and relies on the supervenience of formal and abstract rules to achieve co-operation and consensus and, (2) a co-operative model which gives primacy to relationships and relies on contextual narratives and dialogue—communication—to resolve moral problems. The two different modes of describing the relationship between self and other are essentially two different ethics, the one an ethics of 'universal' rights and duties and 'universal' rational principles, the other an ethics of care, responsiveness, and responsibility. . . . The first ethics is represented mainly by images of opposing positions and hierarchical orderings, while the second is represented mainly by images of communicative and collaborative positions, and replaces images of hierarchy with images of webs, networks, and weavings.

Johnson, in *Moral Imagination: Implications of Cognitive Science for Ethics*, elaborates on these contrasting orientations of morality through a linguistic analysis of various brands of moral argument.

[32] Berry, *The Unsettling of America: Culture and Agriculture*, 47.

Education, then, can neither privilege societal interests nor focus on the whims of the learner, but must seek to understand their interdependencies; it can neither cling to what is given nor relentlessly pursue the limitless imagination, but must explore the transformative possibilities of thought and action; it can neither strive for certainty nor dwell in chaos, but must embrace the complexities of existence; it can neither grasp for the future nor hold only to the past, but must be sensitive to what was and what might be as represented in the context at hand. Education is thus neither this nor that, but dwells in the movement between the two. Just as the capacity for motion is an essential quality of all living things, education is a necessary trait of our living culture.

I am thus in accord with Bruner:

> [A] culture is constantly in process of being recreated as it is interpreted and renegotiated by its members. In this view, a culture is as much a *forum* for negotiating and re-negotiating meaning and for explicating action as it is a set of rules or specifications for action. . . . Education is (or should be) one of the principle forums for performing this function—though it is often timid in doing so.[33]

And so, whether we wish to assume moral responsibility or not, we who are involved in education are also actively engaged in the transformation of our culture—even while we attempt its transmission. This is not to say, however, that through education we should seek to "overcome" the past. Because we are "historical beings," we must rather situate ourselves—seeking, in Heidegger's terms, a "conversing with that which has been handed down," so that there is no "break with history, no repudiation of history, but . . . an appropriation [*Aneignung*] and transformation [*Verwandlung*] of what has been handed down to us."[34] An attitude of listening for that which speaks to us in our traditions—recalling that listening is not a passive attendance but an active participation—is thus demanded.

In sum, a theory of enaction prompts us to work conscientiously toward cultural reform (while acknowledging that we cannot help but participate the cultural transformation). In effect, we must acknowledge the co-implicative structure and the dynamic nature of the place of education. To fail in this task is to be complicit in promoting what Varela et al. describe as "the sense of nihilistic alienation in our culture"[35] and what Charles Taylor has labeled "the malaise of modernity."[36]

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[33] Bruner, *Actual Minds, Possible Worlds*, 123.

[34] Martin Heidegger, *What is Philosophy?* (New York: Twayne, 1958), 71.

[35] Varela et al., *The Embodied Mind*, 253.

[36] Charles Taylor, *The Malaise of Modernity* (Concord, ON: Anansi, 1991).

Section B

Artistry: The Place of the Teacher

> The right stuff is not the same from great teacher to great teacher, so the art of teaching is frustratingly elusive—we know it when we experience it, but as soon as we talk about it, we also know it is constantly slipping away.
> — Timothy Crusius[37]

At the moment of this writing, someone somewhere in this province is planning for or participating in a protest of the government's proposal to reduce spending on education by 20%. Most people are not at all happy about the projected cuts.

Much of the discontent arises from the apparent contradictions that surround the undertaking. At the same time that legislators are lamenting the economic consequences of our grade eight students' lack of academic competitiveness, they are cutting the funding that supports the educational system. It is thus that these politicians are attempting to change their song on schooling: the issue today is inefficiency, eclipsing yesterday's concern for ineffectiveness.

The premier, who assured voters that "He listens. He cares." in the recent election, has promised a series of public forums on the issue. Not content with the format of these forums, teachers and their union have been sponsoring more open debates. Apparently they have also engaged in more devious activities, for it seems that some have discussed the issues (and their own views) with their students. And some may have actually encouraged learners to voice their own concerns publicly.

The government is not happy. The premier and his supporters have broadcast their contempt for both the "irresponsible teachers" and the "disobedient students": "Teachers have no *place* meddling in the affairs of government;" "Their *place* is to teach."

In the middle of this debate about money, then, the issue of *the place of the teacher* has moved into the spotlight, for it is clear that the government authorities are not willing to allow (nor even to recognize) the place of teachers in shaping society. And it seems that the teachers are not going to ignore their place in culture-making.

What is the place of the teacher? That is the topic of this section, and I approach it through an exploration of a metaphor: "teacher as artist."

Teacher as Artist

"The art of teaching" is a common enough phrase. Most often, it is used to draw our attention to the capacity to negotiate the interplay of the subject matter and the lives of learners, and so we usually hear it in arguments that deal with the impossibility of constructing fully scientific bases to teaching and to education. Placed in direct opposition to the scientific mentality in this conception, the artistic is thought to be flexible rather than rigid, sensitive rather than violent, synthesizing rather than analytic, and inclusive rather than reductionist. Art is skill or knowledge acquired through

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[37] Crusius, *A Teacher's Introduction to Philosophical Hermeneutics*, 74.

experience, and it is offered as a counterpoint to the sort of knowledge that comes about through experimentation and rigorous verification.

However, the conceptions of both art and science that underlie the popular notion of "teaching as art" are shallow ones, emerging from a this-or-that mentality that has, since the beginning of the modern era, sought to distinguish between scientific insight and artistic awareness and to separate mechanization from creativity. We thus have need to re-explore the question, What is a work of art?

Gadamer discusses the issue at length. One of his conclusions is that the work of art has a two-fold function. First, it *represents* something that is not immediately present—that is, it re-calls to our senses, it makes present, it stands in for something that is not here. But that is its minor function, for the work of art also *presents*. In Gadamer's words, "a picture is an event of presentation . . . an increase of being."[38] Put differently, the artwork not only points to something that is not at hand, it offers to us something new—something that was not available to our previous seeing or hearing, something that demands that we look again and that we listen anew. The artwork is both an imitation and an interpretation.

A contrast might thus be drawn between the work of art and those visual and auditory products that are intended only to represent—such as photographic snapshots. Unlike a work of art, the photograph aims to make itself invisible. It is not the snapshot, but the image it bears that we notice. The artwork, however, never disappears. Its purpose is not just to represent something else, but to remind us of how we "see," thus challenging the taken-for-granted (the prejudices) that frame our perceptions.

Gadamer thus assigns the work of art an ontological status, in accord with Susanne Langer's contention that

> A work of art expresses a conception of life, emotion, inward reality. But it is neither a confessional nor a frozen tantrum; it is a developed metaphor, a non-discursive symbol that articulates what is verbally ineffable—the logic of consciousness itself.[39]

The work of art, then, is not intended to transmit a message, but to open a space for personal transformation. It encourages the onlooker or the listener to think otherwise, to consider the possibilities that exist at the edge of our awarenesses and that have not yet found form in our common language.

The artist, then, is situated between the actual and the possible, between what is and what might be. The artist must be attuned to—*listening* to—both this and that, for she has consciously thrown herself in the zone of tension between the two. The point, as Madeleine Grumet elaborates, "is that to be an artist is perpetually to negotiate the boundary that separates aesthetic from mundane experience."[40] It is here that the connection to teaching might be made, for both the teacher and the artist are in a place "to challenge the taken-for-granted values and culture that one shares with others."[41] Grumet is thus suggesting that understanding curriculum as an aesthetic text offers us a way of replacing its technical function with a revelatory function.

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[38] Gadamer, *Truth and Method*. 151.

[39] Susanne K. Langer, *Problems of Art* (New York: Charles Scribner's Sons, 1957). 26.

[40] Grumet, *Bitter Milk: Women and Teaching*. 79.

[41] Ibid., 81.

Aronowitz and Giroux[42] use the phrase "transformative intellectual" to describe their sense of the teacher's role. They, like Grumet, Varela et al., and others, point to the inevitability of effecting transformation of one's self, one's students, and one's culture through one's teaching. In spite of our persistent timidity in acknowledging the impact of teaching, the teacher, in pointing and representing, is also and always interpreting and presenting. More importantly, perhaps, and much contrary to the seemingly invisible or non-present role of the teacher in transmission conceptions of education, the teacher is necessarily present and implicated in the subject matter of the classroom. Always and inevitably, she simultaneously re-presents and presents ideas while presenting herself.

A deeper understanding of art, then, can provide us with a profound sense of the place of the teacher. Sadly, it is a sense that has been all but lost in the commodification of art and knowledge in today's consumer culture.

A Way of Putting Things

> I recall a teacher, her name was Miss Orcutt, who made the statement in class, "It is a very puzzling thing not that water turns to ice at 32 degrees Fahrenheit, but that it should change from a liquid to a solid." She then went on to give us an intuitive account of the Brownian movement and of molecules, expressing a sense of wonder that matched, indeed bettered, the sense of wonder I felt at that age (around ten) about everything I turned my mind to. . . . In effect, she was inviting me to extend *my* world of wonder to encompass hers.[43]

Jerome Bruner uses this anecdote to illustrate the disparate educational consequences of two modes of speaking: that which is characterized by certainty, and that which is more tentative. Citing a study in which the sorts of statements teachers made to one another (regarding their subject area knowledge) were compared to the way they spoke to students in their own classrooms, Bruner comments that "the world that the teachers were presenting to their students was a far more settled, far less hypothetical, far less negotiatory world than the one they were offering their colleagues."[44]

Positioned in front of their students, teachers gave little or no sense of the tentative nature of knowledge—a sense which was willingly communicated to colleagues—and, in so-doing, tended to close down invitations to further thought. One might say that these teachers stopped listening to themselves when they entered the classroom. In Bruner's personal history, Miss Orcutt stands out as an exception: "She was a human event, not a transmission device," standing apart from other teachers whose "stances were so off-puttingly and barrenly informative."[45] These teachers endeavored only to represent. Making no deliberate effort to treat the subject matter as something that also presents, that opens, that challenges, that engages, the teachers offered "flat declarations of fixed factuality"—something hardly worth listening to. Bruner sums up:

> To the extent that the materials of education are chosen for their amenableness to imaginative transformation and are presented in a light to invite negotiation and speculation, to that extent education becomes part of . . . "culture making." The

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[42] Stanley Aronowitz and Henry Giroux, "Radical Education and Transformative Intellectuals," in *Canadian Journal of Political and Social Theory* 9 (1984): 48-63.
[43] Bruner, *Actual Minds, Possible Worlds*, 126.
[44] Ibid., 126.
[45] Ibid.

> pupil, in effect, becomes a party to the negotiatory process by which facts are created and interpreted. He becomes at once an agent of knowledge making as well as a recipient of knowledge transmission.[46]

Recalling the earlier discussion of curriculum anticipating, an important component of approaching teaching is the consideration of how one might speak of the ideas to be presented—in much the same way that the artist contemplates the medium for her own creation. The shape of the curriculum is found in the forms of the language used.

From Art Lesson to Math Class

Robert Pirsig points out that the Proto-Indo-European root for both "art" and "arithmetic" is the morpheme *Rt*. (It is also the root of a host of other terms of relevance to the current discussion, including "right," "rhetoric," and "ritual.") Extrapolating from the meanings of its linguistic descendants, Pirsig offers the following definition of *Rt*: "first, created, beautiful, repetitive order of moral and aesthetic correctness."[47]

The idea of considering a mathematical notion as a work of art—of offering to students as something that both represents and presents—then, need not be restricted to the metaphoric plane. A mathematical idea accomplishes just that—but, like the work of art, it can lose its potency through mechanical reproduction or, like the metaphor that dies into literalness as it forgets itself, it might solidify into a mere object through over-familiarity. It is thus that the teacher must be more than a transmissive device; he, whether he chooses to acknowledge it or not, is already interpreting the subject matter. The art of teaching, then, is to be attentive to the interpretation offered, endeavoring to re-organize experience so that it is not lost, not merely assimilated, but perceived freshly.

I do not mean to sound romantic. Rather, I wish to point toward examples such as that of Miss Orcutt whose willingness to share her fascination with something as mundane as water freezing—that is, her unwillingness to be allow her interest to be doused by familiarity—open the possibilities for important (transformative) moments in children's educations. In my own experience, I recall Mrs. O'Brien and Dr. Cristall who, for all their eccentricities, were genuinely animated by the mathematics they taught. It always presented something new for them; it was never offered as something to be mastered, but for what it hinted at, where it led, what it presented. For them, teaching was not a matter of *telling*, but about *listening* for possibilities.

These persons were "transformative intellectuals." I return to this phrase because there remains one issue to address more directly: that these teachers were indeed *intellectuals*. They knew and enjoyed their subject matter, and these qualities provided them with the ability to move beyond the prescribed limits of a curriculum manual. In a sense, these teachers were able to enact the mathematics they taught—the sort of person described by van Manen: "A math teacher is not (or should not be) just somebody who happens to teach math. A real math teacher is a person who *embodies* math, who *lives* math, who in a strong sense *is* math."[48]

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[46] Ibid., 127.

[47] Pirsig, *Lila: An Inquiry into Morals*, 254.

[48] Max van Manen, *The Tone of Teaching* (Richmond Hill, ON: Scholastic-TAB, 1986), 45. (original emphasis)

A person who is only "one page ahead" of the learners is not likely to do much more than fixate on the image and ignore the substance, in effect, to rob learners of the very reason the subject is worthy of study in the first place. This being the case, we would perhaps do well to insist that our teachers of mathematics have some sense of what Hans Freudenthal calls the "phenomenology of a mathematical concept," a phrase he uses to refer to the process of situating the concept in "relation to the phenomena for which it was created, and to which it has been extended in the learning process of mankind."[49] Freudenthal's purpose in articulating this notion (and in providing rather extensive illustration) is to offer the mathematics educator a "didactical phenomenology"—"a way to show the teacher *the places* the learner might step into the learning processes of mankind."[50]

The Place of the Teacher

What is the place of the teacher?

If we are to conceive of her role as that of a transformative intellectual, then she is that artist who is attuned to and who moves back and forth between the collective and the singular, past and future, actual and possible. She is the one who interprets and who, in her interpretation, opens a space for transformation. The teacher's task is thus not merely to re-present and in that objective representation, to make herself invisible. Rather, the teacher is in every way implicated in the subject matter.

But, like any metaphor, "teacher as artist" obscures as much as it illuminates. The artist, for example, is an artist by virtue of the sort of work she produces—and "work" in this context has a very particular meaning. An artwork is a performance, the result of a likely prolonged, but largely invisible, labor. It arrives to its public already formed and it thus masks both its history and the intricacies of the relational web from which it arose. Thus, although the metaphor may well challenge and provoke us to think otherwise about some matters (for example, in addition to bringing the teacher into the teaching, it helps us toward an understanding of "work" that distances it from the repetitiveness of factory-like labor), its usefulness for informing teaching is limited.

For the "work" of the teacher is unlike the "work" of the artist; a teacher's work cannot be "finished" or "performed" in the same way. We do not gaze at the endpoint of teaching (although there is a pervasive tendency to fixate on the consequences of teaching performance: student achievement), but rather at the sustained effort, the mundane day-to-day-ness of life in the classroom. The artist's work is an endpoint—albeit one that is continually transformed through performance; the teacher's work is a labor which never sees its completion.

And the artist's work acquires a certain autonomy. Indeed, we tend to construct clear and distinct boundaries around such work by framing or printing or staging. The teacher's work does not have such boundaries. Lacking these bounds, it becomes difficult to talk about where teaching takes place, even if we are able to agree upon the place of the teacher.

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[49] Hans Freudenthal, *Didactical Phenomenology of Mathematical Structures* (Dordrecht, The Netherlands: D. Reidel Publishing Company, 1983), ix.
[50] Ibid. (emphasis added)

Section C

Pedagogy: Where Teaching Takes Place

> [Talk] of teaching must consist in symbols, metaphors, which signify in a plurality of ways not only what we are doing in the moment but the possibilities of the moment, the negations and affirmations in the moment which open us up to projections beyond the moment.
> — David Denton[51]

Sarah's head is down, face hidden in her crossed arms. She doesn't move. One minute. Two.

Mr. Wallace notices, and he moves toward her desk as soon as he finishes re-explaining a bit of his earlier lesson to another student.

"Troubles, Sarah?"

"I can't do this," comes the muffled reply.

"Let's see what you're working on."

Sarah lifts her head. She's on question five: "Give the reciprocal of each fraction."

"Okay, the reciprocal is the flip, right?" Mr. Wallace utters his question in the tone of a statement.

"Uh-huh."

"So you just flip these. That's easy."

"I don't get *this* one." Sarah points at part C of the question: "12." "It's not a fraction."

"But it's easy to turn it into one." Mr. Wallace points at the dividing line between the numerator and the denominator of another fraction. "Here, what does this mean?"

"That it's a fraction." A statement, uttered in the tone of a question.

"Yes, but what does this line tell you to do?"

Silence.

"To divide, right?" Another one of those telling questions.

"Uh-huh."

"So a fraction is a dividing statement. Here, three over four means three divided by four. . . . So what could we put under the twelve to make a true dividing statement?"

"Twelve?" Almost too quiet to be heard.

"Is twelve divided by twelve still twelve?"

The answer is once again obvious from the tone of the question. Sarah shakes her head and hangs it a little lower. "I don't get this," she whispers.

"Here." Mr. Wallace takes Sarah's pencil and writes "$\frac{12}{0}$" in the spot that's been prepared in her notebook. "What can you put in this box? What will give you twelve?"

"Zero?" A spark of hope in her voice.

"You know that you can't divide anything by zero." A bit of frustration seeps into his words. "What can you divide by that gives you the same number you started with?"

"I don't know."

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51 David E. Denton, "That Mode of Being Called Teaching," in *Existentialism and Phenomenology in Education*, ed. D. E. Denton (New York: Teachers College Press, 1974), 107.

"One." His impatience is now clearly audible. "Twelve divided by one is twelve. So you can put a one in that box." He picks up the pencil for her.

Sarah takes it and writes in the numeral.

"So, what's the reciprocal of twelve?" Mr. Wallace asks, just to be sure.

Sarah's tone is flat. Distant. She has cut her self out of the situation. "Twelve over one."

This is a re-telling of one of the many teacher-student interactions that I have observed over the past few years. It occurred in a grade seven classroom.

The event remains vivid in my memory, and I can still hear Sarah's last response. It was at that point that Debbie, one of Sarah's friends who was listening in from several rows away, interrupted and offered to explain to her the process of finding a reciprocal. Mr. Wallace gladly consented.

The interesting point here is that Debbie's explanation was little different from that of Mr. Wallace. She used the same words, she followed the same sequence, she pointed at the same examples. Yet, for some reason, teaching took place: Sarah "got it" when Debbie explained. As I look back, that reason is clear. Sarah and Debbie were friends, and in the intimacy of their friendship, they were able to listen to one another. That is, not only was Sarah, as a student, able to learn more effectively because she was better able to listen to Debbie, Debbie's teaching was enabled by her ability to listen more deeply to Sarah's statements and to the unworded parts of her speech—answers, questions, and feelings that revealed more than an undeveloped or inappropriate understanding of the term "reciprocal."

This sort of listening relationship was absent between Sarah and Mr. Wallace. Because of its absence there was no space for teaching or learning to happen. The problem did not spring from his limited knowledge of mathematics (although his narrow conception of the discipline may have contributed to his deafness); nor did it arise from a lack of caring for Sarah (their relationship outside of class time seemed to be quite comfortable and friendly).

In this section, I seek to explore the nature of the relationship between teacher and learner—or rather, a possibility for how that relationship might be conceived. The need for this exploration emerges from Gadamer's explication of the conversation (as elaborated in Chapter 1). For him, the conversational relation is a triad involving three elements: you, me, and the subject matter. Having looked at the issue of our relationship to the subject matter already, I turn to an investigation of the you-me part of the conversation.

My contention at the outset of this discussion is that there is little reason to raise the issue of listening in the context of teaching if we are not willing to critically analyze and explore alternatives to the sorts of relationships that typically exist between teachers and learners in the modern mathematics classroom.

Following this chapter's theme of "place," I develop this section around a discussion of the "bounds" of the place where teaching happens. My selection of this polysemous term as the focal point of the discussion is deliberate. In contrast to the convergent tone of the previous section, here I would like to more fully acknowledge the ambiguities, the contingencies, and the complexities of the sorts of relationships that are enacted in the classroom.

bound *n.* **1**: a limiting line ⟨out of *bounds*⟩: BOUNDARY

The *bounds* are the limits that separate this place from that place; the marking of bounds is the first step in transforming a space into a place. It assigns a shape or a form.

In setting the bounds of teaching, we might begin by pointing to the physical bounds of the schoolyard or the classroom, the temporal bounds of the school year or the class period, or the interpersonal bounds of the school population or the class members.

But we must be cautious here. David Denton speaks against our tendency to scribe these sorts of borders, arguing that such delimitation "of teaching constitutes an imposition on that situation, an imposition with normative force: the situation must be made to conform to this definition."[52] One of Denton's purposes in writing is to suggest that there is something indescribable, something that cannot be bounded, that makes teaching what it is. For him, teaching is not a set of actions, nor a role, but a "mode of being." As such, while we may choose to speak of bounds, we cannot impose them. At best they can serve as traces of where teaching has been.

Dwayne Huebner echoes this disdain for our tendency to impose bounds on teaching:

> The closed classroom door can be very deceptive and illusory; it merely hides the inherent communal nature of teaching. The vocation of teaching is living a life in the real world.[53]

Later he adds that "teaching is a way of living, not merely a way of making a living."[54] It is thus, for example, that a teacher's favorite stories about teaching are often situated outside of the bounds of the school or the classroom: in grocery stores, at home, during the summer, after school, with friends, among strangers.

Nevertheless, there is still something to be gained in exploring this issue, but we must first loosen the bounds on "bound." Mathematics, I believe, offers a means of doing this, for "bound" is a notion that has been borrowed and elaborated upon by mathematicians. As with any term that has been co-opted for the purposes of mathematics, it returns to us with a new richness.[55]

In his wide-ranging exploration of issues surrounding artificial intelligence, prominent mathematician Roger Penrose distinguishes between a recursive set with a simple boundary ("so that one can imagine it being a direct matter to tell whether or not some given point belongs to the set"[56]) and a recursively enumerable but non-recursive set which has a complicated boundary—"where the set on one side of the boundary is supposed to look simpler than that on the other."[57] As Penrose elaborates, for such a non-

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[52] Ibid., 101.

[53] Dwayne Huebner, "The Vocation of Teaching," in *Teacher Renewal: Professional Issues, Personal Choices*, eds. F. S. Bolton and J. M. Falk (New York: Teachers College Press, 1984), 19.

[54] Ibid., 29.

[55] I am attempting here to practice what I announced in the preceding section: to use a mathematical idea, albeit metaphorically, to *present* some aspect of the world. To adapt Freudenthal's words, I am trying to show a place where we might step into the learning process of humankind.

[56] Penrose, *The Emperor's New Mind: Concerning Computers, Minds, and the Laws of Physics*, 160.

[57] Ibid.

recursive set, "there is no general algorithmic way of deciding whether or not an element (or 'point') belongs to the set."[58]

It is this sort of bound that enframes teaching and teachers. For those who fall outside the bound, the line of demarcation is a simple one. But for those standing inside—for teachers—it is complex, and the question of whether this act or that issue belongs to teaching is not always easily resolved. Indeed, whether or not the bulk of those activities that we hope fall into the category of "teaching" are actually educative can only be determined by someone outside the bounds—by the learner.

Nevertheless, there is a bound; some actions clearly belong to teaching while others clearly do not. That they cannot always be specified—that no algorithm or definition can help us to decide where teaching begins and ends—does not mean that it lacks bound.

Another context that might help to push out the bounds of "bound" is the attempt to determine the limits of the earth's atmosphere. As my fifth grade science teacher explained, we cannot determine where our airspace ends and where "outer space" begins. The bounds we use tend to be arbitrary, chosen according to the application at hand. Regardless of the haziness of the distinction made, however, it is clear that some objects are within the atmosphere and some are not. The atmosphere of teaching, is similarly ill-defined, and, like the earth's atmosphere, perhaps better discussed in terms of qualities and characteristics than in terms of discernible limits.

Otto Bollnow develops the metaphor of atmosphere in a way that helps to give shape to the sorts of places where teaching might happen. For him, the "pedagogical atmosphere" refers to "all those fundamental emotional conditions and sentient qualities that exist between the educator and the child and which form the basis for every pedagogical relationship."[59] (Regarding his use of the term "atmosphere," then, Bollnow's concern is with the pre-conditions of adult-child relationships that are necessary for the rearing of our children, and not with the "emotional and sentimental undertone" which is often associated when "atmosphere" is used to refer to relationships.)

Bollnow is not just interested in the virtues of the teacher, but with the virtues of the child, and his exploration is thus conducted from the perspectives of both educator and learner. For the teacher, Bollnow identifies and elaborates on the qualities and attitudes of confidence, (reciprocal) trust, love,[60] expectation, patience, hope, serenity, humor, and goodness. Rather than doing an injustice to his work by attempting a summary, I will let this list stand as it is, for Bollnow has identified those virtues which, I feel, are the markers that give shape and form to the place where teaching occurs. They are the proper bounds of teaching, following Heidegger's reconceptualization: "A boundary is not that at which something stops but, as the Greeks recognized, the boundary is that from which something *begins its essential unfolding*."[61] The qualities (boundaries) of the pedagogue as identified by Bollnow thus provide us not just with a sense of the form of teachers' relationships with children, but with an idea of the place

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[58] Ibid., 161.

[59] Otto F. Bollnow, "The Pedagogical Atmosphere," in *Phenomenology and Pedagogy* 7 (1989), 5.

[60] I flag this word to signal my own discomfort in using it—a symptom, perhaps, of the pervasive technical/clinical orientation to adult-child relationships in the contemporary school setting. Nevertheless, encouraged by the examples of curriculum theorist Madeleine Grumet and scientists Maturana and Varela, I have elected to include it.

[61] Heidegger, *Basic Writings*, 332. (original emphasis)

from which they unfold. A basis is thus established for approaching teaching with an orientation to listening. (Indeed, if one were to identify the qualities of a good listener, one would create a list that would be very much the same as Bollnow's, above.)

bound *adj.* **2**: intending to go ⟨homeward *bound*⟩: ORIENTED

Much of the difficulty we experience in trying to talk about the borders of teaching arises not from the fact that they are ill-defined, nor because the bounds of teaching might better be thought of as necessary qualities rather than limiting lines, but because the bounds of teaching are in constant motion. The place of the teacher is constantly being negotiated as those elements that frame teaching move (that is, as society evolves, as the learner learns, etc.) and the place where teaching happens can never therefore be held still.

Saying that teaching is oriented and that it is impossible to pin down the place where teaching happens is not the same as suggesting that teaching has a specific goal (i.e., in the progress-insistent modernist sense of the term). Teaching is intentional; it *takes* place. A more appropriate sense of where teaching is bound is captured by Max van Manen in his elaboration of the notion of pedagogy:

> Pedagogy refers only to those types of actions and interactions intentionally (though not always deliberately or consciously) engaged in by an adult and a child, directed toward the child's positive being and becoming.[62]

The guiding principle of the pedagogue, then, extends beyond the reductive epistemological frame of the modern school. In this way, recalling an earlier citation from Levin, a pedagogical orientation has much in common with a listening orientation:

> In listening to others, accepting them in their irreducible difference, we help them to listen to themselves, to heed the speech of their own body of experience, and to become, each one, the human being he or she most deeply wants to be.[63]

Of course, characterizing the educational intention in this way gives rise to several questions. Is the criterion that teaching contribute to the "child's positive being" an adequate basis to select one's teaching actions? How might we separate our own personal intentions from the pedagogic endeavor? How might we resolve the conflicts between our own pedagogic hopefulness and the child's intentions? The answers to such questions, I believe, are to be found by first understanding that pedagogic intentionality is in no way prescriptive. It cannot determine or serve as a measure for a particular action. Rather, the notion of pedagogy points more toward a way of standing in the world, and it is founded not on a desire for perfection, but on the knowledge that indecision, ambiguity, and tension are inevitable parts of living.

bound *adj.* **3**: under moral obligation ⟨honor *bound*⟩: COMMITTED

There are deliberate moral dimensions to Bollnow's and van Manen's explications of pedagogy; part of their project in writing is to redirect our attention to the

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[62] van Manen, *The Tact of Teaching*, 18.

[63] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 88.

human and relational elements of educating and away from the impersonal and distancing tendencies of modern schooling.

The point has already been made that our identities are affected by historical contingencies and are tied up with social relationships. If we pause to think about the classroom context, it is not difficult to produce a list of the sorts of personal relationships that are either indirectly affected by or directly mediated by the teacher. Most obviously, there is the teacher-child association (the locus of van Manen's "pedagogy"). But the teacher also helps to shape the learner's relationship to knowledge, to others, and to the collective. Conventional schooling, which seems to be founded on an aversion to the topics of morality and ethics, chooses to acknowledge its role in affecting only one of these relationships, the epistemological—and, even there, the system seems non-cognizant of the ontological status of one's knowledge.

Enactivist theorists make a profound contribution to this discussion, a contribution that is perhaps best articulated through their notion of "being and becoming"—on *identity* or *self*hood. Briefly, enactivist theorists join in their rejection traditional (Western) conceptions of the Self (understood in modern terms as a unified, coherent, singular, and insulated agent). Some, including Varela et al. and Charles Taylor, suggest that it is our grasping after the objectified subject that underlies much—and perhaps most—of human suffering. In its place, they offer a constantly evolving self—one that brings with it not only a history of personal experience, but the accumulated cultural and biological histories of the species. "Self," so-conceived, is "a social linguistic construct, a nexus of meaning rather than an unchanging entity."[64]

These thinkers do not wish to dissolve or discard popular notions of self and identity. Their point is not so much that the modern conception of Self is useless (or even destructive), but that it is mis-conceived. In their view, rather than existing as a pre-given transcendent object, the self is *enacted* and *embodied*. It exists and finds its form in bodily action and interaction. And so, enactivist theorists seek to re-awaken our awareness of the active, negotiated, storied, and relational natures—that is, to give a dialogical rather than a monological account—of our selves. As Foucault explains:

> [The] subject should not be entirely abandoned. It should be reconsidered, not to restore the theme of an originating subject, but to seize its functions, its intervention in discourse, and its systems of dependencies.[65]

To illustrate this point, we might return to Gadamer's investigation of the role of a work of art. For him, art is art in the fact that it can become an experience that changes the experiencer. Gadamer uses the concept of "play" to describe this experience, stating that "play fulfills its purpose only if the player loses himself in the play."[66] There is a forgetting of the Self in play (and, correspondingly, in experiencing a work of art), and a subsequent returning to subjective awareness—but, upon return, the Self has been transformed. (The topic of play will be more thoroughly explored in the next chapter.)

If we are to regard the educator as a transformative intellectual, one who endeavors to open these places for play, then the ethical implications are clear: the

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64 Anthony Paul Kerby, *Narrative and the Self* (Bloomington, IN: Indiana University Press, 1991), 34.
65 Michel Foucault, *Language, Counter-Memory, Practice: Selected Essays and Interviews*, trans. Donald Bouchard and Sherry Simon (Ithaca, NY: Cornell University Press, 1981), 137.
66 Gadamer, *Truth and Method*.

teacher "shares in, but does not cross the boundaries of the other person's being."[67] Pedagogy—that acknowledgment of the moral status of the teacher-child relationship, or of any relationship, for that matter—is an area of scholarly inquiry that must be brought into our research in mathematics classrooms. Unfortunately, it is a notion that is notable in its absence from much of the research in mathematics education, where methodology is substituted for care and technology replaces personal contact.

bound *adj.* **4**: fastened by or as if by a band ⟨tightly *bound*⟩: RELATED

"The practical consideration for a teacher is that he or she must believe there is a pedagogic way of being with children that set a teacher-child relationship apart from any other kind of adult-child connection."[68] It is a relationship *sui generis*.[69]

Bollnow, Spiecker, and van Manen have elaborated on the *pedagogical relationship*, and so extensive explication will not be attempted here. For the current purposes, a few additional statements will suffice to provide a flavor of this relationship:

> In pedagogical situations the adult and the child do not just happen to be in the same spot; rather, they are together in a special way. They are together in an interactive unity that constitutes a relation, a pedagogical relation.[70]

The theme that I have used to structure this chapter is *place*, or more specifically, how particular places come to be. An understanding of relationships is central to this discussion. Like humans, the identities of places are established relationally—that is, by the sorts of relationships that they invite, define, or facilitate, and which, in dynamic reciprocal action, continuously re-form their landscapes. A place is a locus, an ecology.

The pedagogical relationship is a place—one that is bounded by particular virtues and one that has as its reason for existence the education of the child. It is where teaching takes place, for within this relationship there is the possibility for setting aside one's agenda, one's desire to predetermine outcomes, one's drive to control. Within this relationship, one is able to attend to possibilities, and not merely to the actualities that are imposed by conventional curriculum-making and instructional practices.

The point here is that, as we seek to determine where teaching takes place, as we explore the *bounds* (in all its polysemous splendor) of teaching, we return to relationships—to the everydayness of life. The place that teaching happens is not found between bells or classroom walls, it is in the immediacy and the intimacy of the interactive unity of one person with another.

Teaching is a fabric of relationships. It is an identity. These ideas are implicit in notion of *pedagogy*, and it is thus that an understanding of the special relationship between teachers and learners provides us an opportunity to heal the modernist separation of life and work; it challenges the belief that we can live differently and apart from the way we make our living. It allows us to act on Wendell Berry's warning: "If we do not live where we work, and when we work, we are wasting our lives, and our work too."[71] It

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[67] Ben Spiecker, "The Pedagogical Relationship," in *Oxford Review of Education* 10 (1984): 203-209, 204.
[68] van Manen, *The Tone of Teaching*, 52.
[69] Spiecker, "The Pedagogical Relationship."
[70] van Manen, *The Tone of Teaching*, 72-73.
[71] Berry, *The Unsettling of America: Culture and Agriculture*, 79.

is thus that, when I say that I am a teacher, I am not saying how I make my living, but how I live. I am announcing where I take place.

To Listening

A colleague[72] recently conducted a series of interviews with several high school English students. The intended focus was a unit of study that had just been completed. However, even though he never intended it to be a central issue, the discussion inevitably turned to the teacher. Asked what it was that separated her from their other instructors, most students responded immediately that it had to do with the way she listened.

What is interesting here is not that this teacher might have been paying more attention to what these students had to say, but that these students independently—yet almost unanimously—selected the idea of *listening* to describe their relationship with her. The fact is that only a small part of her time was taken up in attending to their spoken words, so the listening that these students were pointing to was much more than the complement of speaking. I suspect that, had they the word to use, they would have described their relationship with this teacher as pedagogical. These students knew her concern was genuine, that she lived as a teacher, that there was a particular intimacy between she and they. In this case then, pedagogy and listening were synonymous.

It has been interesting over the course of this study to note how such relationships tend to be described in the language of the auditory rather than the visual: we talk of tone, rhythm, harmony, resonance, attunement—*listening*. These terms speak not just to the interactive unity of the teacher and child, but to their harmonious situation within the world. In contrast, it is when pedagogical concern is wanting that we turn to the visual and speak of perspectives, views, supervision, surveillance—*watching*.

This contrast is an important one. A teaching founded on seeing is a teaching that stands apart from students; it positions itself at the edge of the classroom so that it can oversee all that happens. In this conception, the "good" teacher is the one with "eyes in the back of his head." A teaching founded on listening places itself in the midst of events as a full participant. Like the "good" supervisor, the listening teacher is aware of what is happening outside of her visual field . . . but not because she maintains a silent vigil. Rather, she is attuned to the rhythms of the classroom.

The last word in this quest to determine where teaching takes place, then, is about listening. Perhaps our frustration in locating this place—and the inability of conventional educational theory to define or manufacture it—is that we have been *looking* for it. There is much to be said for turning formal education on its ear and *listening* for it instead. As Levin explains:

> Good listening draws out, educes, the child's readiness for autonomy; and succeeds because it is a means that is consistent with, in harmony with, its intended end. In the education of children, such consonance is absolutely essential. One can *hear* its presence and absence.[73]

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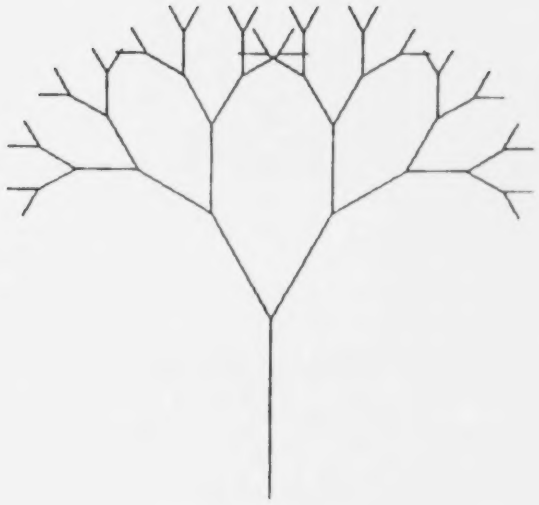
[72] I am indebted to Dennis Sumara for this anecdote.

[73] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 154. (original emphasis)

Chapter 4

ALL EARS
Cognition



Mental activities . . . each draws its metaphors from a different bodily sense. . . . Thus, from the very outset, in formal philosophy, thinking has been thought of in terms of *seeing*. . . . The predominance of sight is so deeply embedded in Greek speech, and therefore in our conceptual language, that we seldom find any consideration bestowed on it, as though it belonged to things too obvious to be noticed. . . . [But] if one considers how easy it is for sight, unlike the other senses, to shut out the outside world, and if one examines the notion of the blind bard, whose stories are being listened to, one may wonder why hearing did not develop into the guiding metaphor for thinking.

—Hannah Arendt[1]

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[1] Hannah Arendt, *The Life of the Mind* (New York: Harcourt, Brace, Jovanovich, 1978), 110-111.

Section A

Knowing

What is *my* answer to the question of the nature of knowing?
I surrender to the belief that my knowing is a small part of a wider
integrated knowing that knits the entire biosphere or creation.
— Gregory Bateson[2]

How do people learn? What does it mean to *know* something? What sorts of experiences lead to changes in behavior, attitude, and conceptualization? These questions—or, more precisely, our answers to these questions—frame our actions as researchers, teachers, and learners.

Historically, the perspectives on learning and knowing that have risen to prominence among educators have been diverse and (seemingly) disparate. Three schools of thought that have had significant impacts in the field of educational psychology are the currently disparaged but still visible *behaviorism*, the now pervasive *cognitivism*, and the persistent *humanism*. Briefly, *behaviorism* concerns itself primarily with the observables of cognition; notions such as thought and emotion are defined in terms of visible actions or responses. For *cognitivism*, the predominant (although not exclusive) orienting metaphor is "brain as computer" and, as such, this discourse system tends to be pre-occupied with knowledge structures, information processing, and decision-making activities. *Humanism* refers to both a psychological and a philosophical orientation, and is centrally concerned with those characteristics that are thought to make us most human.

A quick review of the texts and journals in the field reveals that the *cognitivist* framework is by far the most prominent and broadly accepted by educational theorists. However, this circumstance can hardly be taken to mean that there is wide-spread agreement among cognitivist theorists on the consequences of employing a computer metaphor for the thinking part of us. In particular, two very different perspectives, both of which can be traced to the emergence of Descartes' Rationalism and the consequent Empiricist movement, have found the "mind as machine" notion to be a powerful tool in helping to explicate their positions. These two branches of cognitivist thought I will refer to as Realism and Representationism.

Realist theories are those materialist epistemologies that favor facticity and by which knowledge is regarded as actual or objective bits of information. Whether discovered or created, these bits are treated as though they have a substantive existence: ideas are to be *grasped* and *held*, facts are *cold* and *hard*. Communication is a matter of passing these knowledge bits from one person to another, and the critical aspect of human interaction is thus the selection of the words that are to contain and carry the knowledge. "Thinking," correspondingly, is a matter of processing newly-inputted information by moving it about along neural networks and re-organizing it into new and increasingly complex patterns.

In educational circles, this orientation is popularly known by the monikers of "the acquisition model" and, more depricatingly, "the banking metaphor." It is closely associated with a model of communication that has been described and critiqued by

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[2] Bateson, *Mind and Nature: A Necessary Unity*, 93.

Michael Reddy[3]—the "conduit metaphor." In the conduit framework, communication is understood to occur as a speaker (sender) packages his thoughts into word-containers and sends them through some sort of conduit (e.g., the medium of spoken language) to a receiver who extracts the intended meaning. As will be developed in the next chapter, these are the orientations to knowledge, cognition, and communication that underlie the (predominant) transmission model of teaching.

Bruner notes that, in Western cultures at least, we seem inclined toward Realism: "At our most unguarded, we are all Naive Realists who believe not only that *we* know what is 'out there,' but also that it is out there for *others* as well."[4] Nevertheless, critiques of the orientation have been extensive and condemning, focusing on its inappropriate formulations of both information and interaction. However, the underlying notion of "mind as computer" tends, for the most part, to slip past the criticisms. In fact, most opponents use the same metaphor.

Such is the case with Representationist theorists, who focus not on facticity, but on the ideal of consciousness. Continuing with the project of Descartes, they deny the possibility of knowing the world in any direct way. Building on the Cartesian belief that knowing and thinking involve the development of increasingly accurate representations of the world, the Representationists use metaphors of theory-making and knowledge construction. The resulting emphasis on structures, strategies, and schemas is heavily influenced by our understanding of computers, as evidenced by references to processing, input and output, retrieval, and programming in their accounts of cognition.

The common ground of Realist and Representationist accounts extends beyond their reliance on computer metaphors. Both, for example, are predicated on the notion that learning, while perhaps not smooth, is nevertheless linear and cumulative. And both are founded on the belief that the realm of the mental is utterly distinct from the realm of the physical. Knowing is a matter of the former.

This then is the form assumed by a dualistic mode of thinking about thinking: On the objectivist side, cognition is a matter of employing the senses to extract knowledge bits from the universe. In this way, one comes to know the world as it is. On the subjectivist side, the learner does not come to know *the* world, but creates *a* world. Cognition is thus understood to be a process of autonomous "theory-making" by which the individual develops increasingly accurate (but inevitably unique) representations of the universe. In both cases, a foundational premise is that one's thought or knowledge is valid only insofar as it reflects or corresponds to the pre-given universe.

A Note on "Authority"

The recent movement in mathematics education away from objectivist to subjectivist theories of learning (e.g., from "banking"-related to "constructing"-based metaphors) has been, in effect, a movement from the privileging of an external authority—i.e., pre-given or pre-established fact—to the privileging of an internal one—i.e., the learner's emerging conceptions.

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[3] Michael J. Reddy, "The Conduit Metaphor—A Case of Frame Conflict in our Language about Language," in *Metaphor and Thought*, ed. Andrew Ortony (New York, Cambridge University Press, 1979).
[4] Bruner, *Actual Minds, Possible Worlds*, 65.

The difficulty here is not the location of the authority but that, in both cases, the authorities are "monological" (borrowing Charles Taylor's[5] term). In the more traditional conceptions of education, the monologic authority is the assumed-to-be knowable *real* world. The teacher, the program of studies, and the textbook, as depositories of knowledge of this world, are thus granted primary authority in the classroom. In more child-centered perspectives, the monologic authority is found in the learner's own subjective conceptions—and in this way, a learner's actions can never be considered "wrong" (because they can always be justified in the world that the individual has constructed).

However, as Taylor demonstrates, there can be no such single authority. Rather, issues such as "what we know" and "who we are" are *dialogical*; they are negotiated through our interactions with one another within the context of our culture. That culture, in turn, is negotiated within the wider contexts of history, civilization, and environment.

A Step Toward a Middle Way: Constructivism

Ernst von Glasersfeld[6] tells us that the cognitivist's link between knowledge and reality is one of the distinguishing characteristics of both traditional views of epistemology and conventional orientations to cognitive psychology. Founding his own work on the theories of Vico and Piaget, von Glasersfeld critiques this taken-for-granted relationship which he describes as "always seen as a more or less picturelike (iconic) correspondence or match."[7] Such matches are the foundation of conventional notions of *truth* (i.e., representations that accurately reflect the way things really are). It is at this point—that is, the quest for such truths—that the epistemological orientation of constructivism[8] departs from traditional perspectives.

Briefly, constructivist *knowing* (a term that is used in contrast to "knowledge," in part to emphasize the dynamic nature of one's conceptualizations) is re-cast as "a search for fitting ways of behaving and thinking" rather than the conventional "search for an iconic representation of ontological reality"[9] (i.e., facts[10] or truths). It makes little sense, it is argued, to speak of *representations* of a reality that, as even Descartes acknowledged, is unknowable and inherently inaccessible. Rather, following Vico, constructivists consider knowledge to be a human construction that is to be evaluated according to its fit with the world of human experience. "Representation" is thus redefined by constructivists as "not the mental representation discussed in cognitive science . . . but, rather, the process of transforming the contents of consciousness into a public forum so that they can

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[5] Taylor, "The Dialogical Self."

[6] Ernst von Glasersfeld, "An Introduction to Radical Constructivism," in *The Invented Reality*, ed. P. Watzlawick (New York: Norton, 1984).

[7] Ibid., 20.

[8] Constructivism—like feminism, postmodernism, Marxism, and a plethora of other -isms—has been subject to a wide range of interpretations. Not wishing to get bogged down in the intricacies that separate these theories, I will be using the term to refer specifically to the theory of "radical constructivism" as elaborated upon by von Glasersfeld and others. Where a differing perspective is referenced, it will be flagged with an appropriate modifier.

[9] Ibid., 39.

[10] Denton, in *Gaia's Drum: Ancient Voices and Our Children's Future*, points out that "fact" is not a modern concept; rather, our modern use of the term is a distortion of earlier uses. Originally a fact was a deed (or the doer of the deed), not a building block of reality. As such, the constructivist movement might be characterized as "a return to the facts," in the ancient sense of the phrase.

be stabilized, inspected, edited, and shared by others."[11] It is thus that the criterion of *truth* (in the modern sense of matching with an objective reality) is abandoned in favor of a requirement of *viability*.

The meanings of *fit* and *viability* here are the same as those used by evolutionary theorists, and it is at this point that we note Piaget's influence. In biological terms, the criterion for an organism's survival is its fitness—that is, the environment inexorably eradicates whatever does not fit the constraints it imposes. The logic here is not *prescriptive* (whereby some external agency pre-determines the qualities necessary for survival), but *proscriptive* (whereby an entity remains viable so long as it is not annihilated). At best, then, the environment is responsible for extinction, not for existence.

Put differently, in Post-Darwinian theories of evolution, there is no causal link between an organism's survival and the world, since the theories are not based on the idea of cause-and-effect but on the principle of constraints. As this notion is applied to discussions of cognition, we arrive at a very pragmatic orientation to knowing: the sole criterion for the existence of a conceptualization is that it be feasible within a given setting. It must "work," in that it must provide a basis for appropriate (in the sense of non-self-annihilating) action. (This quality also serves as the pragmatist measure of *truth*.)

The emphasis on action is critical here, for unlike the epistemological orientations it aims to supplant, constructivism does not draw a rigid distinction between mental and physical "objects." As Piaget puts it,

> the empiricist tradition . . . regards knowledge as a kind of copy of reality and intelligence as deriving from perception alone. . . . As if there were nothing more in mental life than sensation and reason—forgetting action![12]

Body and mind cannot be considered as distinct. In the constructivist conception, all knowing is founded on bodily action or sensation; conversely, the evidence of one's body of knowledge is found in one's behavior. It is thus that we arrive at the two main principles of constructivism, as identified by von Glasersfeld:

> (a) knowledge is not passively received but actively built up by the cognizing subject; (b) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality.[13]

Again, it must be emphasized that these principles mark a radical departure from traditional notions of knowledge, truth, objectivity, and reality. As von Glasersfeld puts it, "instead of an inaccessible realm beyond perception and cognition, it now becomes the experiential world we actually live in."[14] It is thus that, as conventional understandings of knowledge and truth are set aside, constructivism must posit itself as merely an

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11 Elliot W. Eisner, "Forms of Understanding and the Future of Educational Research," in *Educational Researcher* 22 (1993): 5-11, 6.

12 Piaget and Inhelder, *The Psychology of the Child*, 28-9.

13 Ernst von Glasersfeld, "Constructivism in Education," in *The International Encyclopedia of Education, Supplementary Volume*, eds. T. Husen and T. N. Postlethwaithe (Oxford, GB: Pergamon Press, 1989), 162.

14 Ernst von Glasersfeld, "An Exposition of Constructivism: Why Some Like it Radical," in *Constructivist Views on the Teaching and Learning of Mathematics* (a monograph of the *Journal for Research in Mathematics Education*), eds. R. B. Davis, C. A. Maher, and N. Noddings (Reston, VA: National Council of Teachers of Mathematics, 1990).

hypothesis, not an absolute truth—a situation that, from a modern perspective, seems paradoxical.

With the constraining demands of a match with the real world erased, constructivism is often misinterpreted as suggesting that the cognizing agent's conceptualizations emerge freely and in completely arbitrary ways—and, thus, that the theory tends toward solipsism. In fact, constructivists argue quite the contrary: that, according to the requirement of fit-ness, the individual's interpretations of and abstractions from experience are shaped by the learning context and, in particular, by the social milieu. The needs and opportunities for collaboration and communication thus demand that the person's "knowing" fall within the bounds of the social setting's construction of normalcy. The role of the environment, albeit proscriptive rather than prescriptive, thus remains central to the construction of one's understandings.

One Step Closer to a Middle Way: Enactivism

Although offering a different orientation to cognition, constructivism appears to share many of the shortcomings of more traditional theories of knowing. In particular, in attempting to position itself as "merely" an epistemology (in contrast to a philosophy, which also addresses matters of ontology), constructivism has situated itself in that modernist niche that attempts to disregard issues of moral and ethical import. Existence, in this frame, is an unproblematic given.

This criticism is not a minor one for, as has been illustrated by the pervasive acceptance of constructivism within the field of mathematics education (which also attempts to locate itself in the same neutral corner), it has allowed researchers to re-orient themselves theoretically without compelling them to more critically examine the nature of their task. The modernist ideal of "How can we do this better?" has thus continued to eclipse the perhaps more urgent "Why are we doing this?" (It is interesting to note, in retrospect, how these two questions framed my own schooling experience. The former guided my actions as a teacher; the latter has figured prominently in my learning.)

A second point of critique is constructivism's inability to account for cultural knowledge. Briefly, in denying the possibility that knowledge exists "out there," and in failing to address the issue of human interactive capacities, constructivists are compelled to locate all knowledge within individual knowers. The critique of the theory's tendency toward solipsism, although perhaps extreme, is thus in many ways justified. Constructivism is a theory of how the individual comes to know—and "individuals," for the most part, are understood in modern isolating and insulating terms.[15]

A third critique is suggested by Varela et al., who embrace much of Piaget's work. However, in it they note "an interesting tension": Piaget is "an objectivist theorist who postulated his subject matter, the child, as an enactive agent, but an enactive agent

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[15] This particular criticism is, perhaps, more appropriately leveled against more "trivial" versions of constructivism. As Les Steffe (personal correspondence) has pointed out, radical constructivism assumes that the individual is an interactive organism: "A construction only follows on from interaction—without interaction there is no construction."

However, it remains the case that the issue of our tremendous interactive capacities is simply not one that is addressed by even radical versions of constructivism—not because the theoretical framework offers no insight into this phenomenon but because, with the particular focus of constructivism on how the individual comes to know, such issues as communication and collective knowledge are peripheral.

who evolves inexorably into an objectivist theorist."[16] This tension, they suggest, arises because Piaget did not have occasion to question the existences of a pre-given world, an independent knower, and a pre-given and logical endpoint to the knower's cognitive development. These "objects" were assumed in his theories.

Putting these last two points of critique in different terms, constructivist theory retains at least two Cartesian ideals: the separation of the individual from the environment and the belief that developmental endpoints are not only desirable, but knowable (and, in this case, pre-determined). What is not clear, or what is perhaps lacking, is an account of the continuing impact of the independent individual knower on the external world. It is here that ecological and enactivist thought can inform constructivism.

Briefly, one might describe enactivist theory as being concerned more broadly with the collaborative construction of a subjective world. As such, the concern is focused on the dialogic interface of mind and society rather than on a dynamic mind seeking to make sense of an ontological given. In response to the two shortcoming of Piagetian constructivism, then, enactivist theorists offer a perspective on cognition that involves both becoming part of an ongoing existing world *and* the shaping of a new one. Acknowledging the role of the individual in affecting this world's form, as discussed earlier, effectively pushes enactivist thought into the realm of the moral. And by addressing the issue of how the world has come to be as it is, enactivist thought places itself in the space of ontology.

It is also this premise of co-emergence or deep interdependence that enables enactivist theorists to sidestep the solipsistic quagmires that seem to tug at constructivists, in spite of their best and most persistent efforts. In positing the autonomous knower—or, more precisely, the goal of the autonomous knower—constructivists create for themselves the problem of accounting for the interactive capacities of ostensibly independent beings. How is it that we can communicate so freely and effectively if we are isolated from one another? Enactivists, in contrast, take mutual affect and historical effect as a fundamental tenets. (It is thus that these theorists tend to align themselves more closely with Buddhist and Taoist thought than with the Judeo-Christian and Aristotelian traditions.)

The point can be illustrated by reference to a particular implication of constructivism. As stated by von Glasersfeld, the individual's "experiential world is constituted by the knower's own ways and means of perceiving and conceiving, and in this elementary sense *it is always and irrevocably subjective*."[17] In this formulation, the possibilities for "joint action" or "intersubjectivity," as elaborated upon by Merleau-Ponty, Gadamer, Maturana and Varela, and others, are effectively discounted. The bounds of the individual, as constructed by constructivists, are impassable. For enactivists, such bounds—while "real" in the sense that they have become for us an experiential reality—are illusory. (With regard to schooling, as will be noted in the next chapter, the consequence of the constructivist notions of subjectivity and intersubjectivity place severe constraints on discussions of what teaching might be.)

This most fundamental distinction between enactivist and constructivist theorists can be illustrated with reference to the notion of "self." Piaget insightfully departed from conventional wisdom by suggesting that one's identity was constantly undergoing

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16 Varela et al., *The Embodied Mind*, 176.

17 Ernst von Glasersfeld, *Aspects of Radical Constructivism and Its Educational Recommendations*, presented at the Seventh International Congress for Mathematics Education (Quebec City, PQ, August, 1992), 2. (emphasis added)

transformation and so the process of change was something that was continuously happening to the self. Enactivist theorists take this one step further: the transformational process is not something that *happens to* the self, it *is* the self. Gallagher phrases this idea in terms of narration: "The 'self' . . . is not a totalized self-identical essence, but a 'self-narrative,' a self-process which never stops being a process in play."[18] "Play" (which is the topic of the third section of this chapter) is used here polysemously and is intended to call to mind a movement which, in Gadamer's words, "has no goal that brings it to an end; rather it renews itself in constant repetition."[19] Within play, there is the possibility for a setting aside of subjective awareness and for coupled action, as illustrated with the examples of the performance of a piece of music, the conversation, the staged production, and the game—each of which we commonly describe in terms of play. An important further note is that, in each case, the phenomenon exists only in the playing. So it is with the (enacted) self, a self-in-process, which might be understood as merely another level of (inter-)play.

The self, then, is defined as a network of relationships, and so, as histories, contexts, and participants vary, identities (or, in Maturana and Varela's terms, structures) change. (This idea is closely aligned with the postmodernist contention that we do not "don different masks"—that is, behave differently—as we move from one setting to another. Rather, different selves are enacted: we change.) Our identities do, however, retain an integrity as we move from one setting to the next, held together by particular habits, language (patterns of acting), stories (narratives), and other knowings.

From the Formulated to the Unformulated

Returning to the issue of knowledge and knowing, Charles Taylor[20] provides us with a useful distinction between two orientations to human action: the *formulated* and the *unformulated*. Traditional epistemologies have focused exclusively on the former—on established, validated truths, or on the quest for such truths. This emphasis springs, at least in part, from the rigid division between cognizing agent and world which requires some sort of intermediary step (in this case, formulated representations) to bridge the gap.

With enactivism, the emphasis shifts to the unformulated whereby, in Maturana and Varela's terms, every act is an act of cognition: "to live is to know (living is effective action in existence as a living being)."[21] This notion is founded on the belief that we are not apart from, but "coupled" to our situation or context. As such, we are constantly and inevitably enacting our knowledge; we are continuously knowing, as determined by our structures and our situations. However, much of this knowing is not, and may never have been, formulated in explicit terms. Much of what we do and know, in other words, is unformulated: we just do it, we just know it. It is thus that, as mentioned earlier, the measure of one's knowledge shifts to effective action in a specific context—and these actions may or may not be (but likely are not) subject to conscious awareness. Knowing is doing, and all doing arises from a rich and ongoing history of structural coupling with a complex and active environment.

It is important to emphasize here that "formulated-unformulated" is not presented as a *this-or-that* sort of dichotomy. These are, rather, complements of one another and

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[18] Gallagher, *Hermeneutics and Education*, 51.
[19] Gadamer, *Truth and Method*, 103.
[20] Taylor, "The Dialogical Self."
[21] Maturana and Varela, *The Tree of Knowledge*, 174.

thus cannot be separated. Nor would we want to do so: our formulations continually emerge from our unformulated actions, even while they fade into such action in a process that might appropriately be compared to the gradual literalization of a metaphor.

As we move to a more specific discussion of the schooling context, the implications become profound, for we can no longer focus our educational efforts on the formulated pre-givens of programs of study or institutional structures. Rather, our attention inevitably must move to emergent understandings and to offering occasions for *play*.

It is interesting to note, as I end this section on the varied perspectives on knowledge, that the alternative offered by enactivist theorists bears many of the same characteristics and draws many of the same conclusions as behaviorist and humanist schools of psychology—orientations that have been completely overshadowed by cognitivism.

Section B
Understanding and Meaning

> Thinking begins only when we have come to know that reason, glorified for centuries, is the most stiff-necked adversary of thought.
> — Martin Heidegger[22]

Most often, the words *understanding* and *meaning* refer to mental processes or states. They are defined in terms of an unproblematic dualistic relationship between the knowing subject and the known object. In modern epistemological terms, these *states* are often described in terms of adequacy of mental representation, where the representation is a third thing that mediates between separated subjects and objects.

By describing understanding and meaning as mental or intellectual operations—that is, by locating them in the mind—we have transformed the processes into objects in and of themselves. They have become the desired goals of learning: *states*. Moreover, the criterion for such understandings and meanings is *state-ability*—that is, the ability to provide definitions, consequences, or results, in rigid and (pre-)formulated terms. It is thus hardly surprising that the word "formal" is often used as a modifier to "education" when we speak of schooling.

These *static* orientations to understanding and meaning are common to both Representationist and Realist perspectives on knowledge, and it is they that undergird the formal testing regime of conventional school mathematics. It is also they that contribute to the perspectives on education in which listening skill is regarded as a necessary capacity of the learner and not of the teacher, for if knowledge is to be measured through re-presentation of one's sense of pre-formulated terms, all that is needed to assess understanding is an appropriately constructed template.

But if the measure of understanding is something different, and if we are willing to acknowledge that the ability to *re-state* "formulated meanings" cannot always be taken as an indication of meaningfulness, then we are compelled to develop different understandings of "understanding" and alternative meanings for "meaning." These then are the goals of this section: first, to draw on some current theoretical work in mathematics education that may offer possibilities, and then, to develop a preliminary understanding of their immediate implications for teaching.

Understanding Understanding

Von Glasersfeld has offered us a means of moving beyond conventional state-ic conceptions of understanding by providing a constructivist definition of the term: understanding is a continuing process of organizing one's knowledge structures.[23] So understood, understanding is dynamic and necessarily caught up in a web of meanings (which, to the teacher's perspective, may or may not be appropriate).

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22 Martin Heidegger, *The Question Concerning Technology and Other Essays* (New York: Harper Torchbooks, 1977), 112.

23 Ernst von Glasersfeld, "Learning as a constructive activity," in *Problems of Representation in the Teaching and Learning of Mathematics*, Ed. Claude Janvier (Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1987).

This redefinition of understanding, however, offers only a first step in re-thinking the notion, for at least two elements of more traditional interpretations remain unchallenged. First, the subjective constraints on understanding continue to prevail; second, a focus on the formulated aspects of one's knowing is retained.[24] Both of these shortfalls are overcome by enactivist theorists in the suggestion that cognition might more productively be understood in terms of adequate functioning in an ongoing interactive world. First, as personal identities are re-interpreted to be emergent, relational processes rather than self-creations, understanding is placed in the realm of interaction rather than subjective interpretation. The possibility is thus opened for shared, rather than merely subjective, understandings. Second, those understandings that are enacted in our moment-to-moment, setting-to-setting movement are acknowledged.

Constructivists, in critiquing the conventional state-ic orientation to understanding, have argued that understandings are founded on a history of experience that is subject to continual re-interpretation. Since each person's background is different, each subject's understanding of a given phenomenon, word, or mathematical concept is necessarily unique. For the constructivist, then, when we speak of personal knowledge, we must speak in terms of compatible rather than shared understandings.

But from an enactivist perspective, whereby "understanding" is discussed in terms of effective action rather than conceptual structure, and whereby words and concepts are interpreted as patterns of acting, shared understandings are quite possible.[25] Our social natures—that is, our capacity to function harmoniously—is evidence of such phenomena in action. Understandings are thus not merely dynamic, they are relationally, contextually, and temporally specific. As one moves away from a particular situation, one's understandings, as revealed in one's actions, may change dramatically. And so,

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[24] To elaborate briefly on this point: In current discussions on the topic of mathematical understanding, a distinction tends to be drawn between "conceptual" and "procedural" understandings (to use the terms developed by James Hiebert in *Conceptual and Procedural Knowledge: The Case for Mathematics*, Hillsdale, NJ: Lawrence Erlbaum). In similar veins, Richard Skemp (in *The Psychology of Learning Mathematics*, Baltimore: Penguin Books) uses the terms "relational" and "instrumental" and Anna Sfard (in "On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin," in *Educational Studies in Mathematics* 22 (1991), 1-36) talks about "structural" and "operational." These terms and categories have provided us with a means of talking about the tension between the abstract nature of mathematics and the algorithmic focus of many conventional classrooms. They thus represent valuable contributions to discussions of mathematics learning and teaching.

However, all of these categories or forms of understanding focus on the formal and formulated dimensions of the phenomenon (i.e., both "relational" and "instrumental" understandings tend to be interpreted in terms of the state-ed or the observed). The enacted and unformulated dimensions of our knowing tend to be overlooked.

by virtue of the fact that both conceptual and procedural

[25] The phrase "shared understandings" should not be interpreted in terms of "sameness of internal (or subjective) formulation/construction/conceptualization/idea/representation." Rather, our (largely unformulated) shared understandings point to the possibility of (and are revealed through) our joint and harmonious actions. To suggest, for example, that we share an understanding of "dog" is not to say that we would, regardless of situation, agree on its meaning. Rather, it is intended to suggest that, in the specific context in which "dog" arises, our intersubjective action is not "de-railed."

An interesting side-note is that the sorts of fluid interactions that emerge from our sea of shared understandings can also be the source of considerable unease and great mystery for persons who are unfamiliar with such patterns of behavior. Numerous accounts of autistic persons, for example, report on the difficulty that these individuals have in responding appropriately to the social cues—i.e., the shared understandings—that most of us take for granted. Many of these persons, in fact, attribute our capacity for such joint (coupled) action to extrasensory perception. (See Sacks, "A neurologist's notebook: An anthropologist on Mars." *The New Yorker*, 27 December 1993, 106-125.)

while understandings might be shared during moments of interactive unity, they inevitably diverge as the participants come back to their selves.

The critical departure from constructivism here is not the semantic move from compatibility to sharing, but the more foundational shift from formulated (the state-able) to unformulated (the enacted). By way of practical example, a learner's understanding of the commutativity of addition might just as well be revealed in a setting where she is manipulating a set of wooden blocks as in a situation where she is encouraged to re-present her interpretations by vocalizing her thinking. The essential point here is that both the physical manipulation and the vocalization are critical elements to cognition. As Vygotsky phrased it,

> Children solve practical tasks with the help of their speech, as well as with their eyes and hands. This unity of perception, speech and action, which ultimately produces internalization of the visual field, constitutes the central subject matter for any analysis of uniquely human forms of behavior.[26]

Or, to paraphrase Maturana and Varela, every act is an act of cognition.

Such enacted understandings are bodily, and, if Mark Johnson's[27] thesis that our linguistic capacities are founded on metaphorical extensions of bodily experience can be trusted, these understandings serve as a sort of repository for the knowledge that underpins more formulated conceptions. (The difficulty here, however, is that with the current emphasis on formulated knowledge in mathematics education, many concepts tend to be covered without regard for the experiences that might support them.) They are a part of our acting in the world—an acting that "understands" the difference between a single or a pair of raised fingers before it can count, an acting that "understands" a sequence of two perpendicular cuts produces four pieces before it realizes the process is multiplicative. These are understandings that are aspects of the body's doing, and are thus conditioned by that which is encountered in moving through the world. The bulk of our understandings fall into this category, with only a small portion ever coming to formulation. In this sense, we simply have no (formal) understanding of the extent of our (enacted) understanding. The role that the body plays in this formulation is not unlike the role played by Freud's "subconscious,"[28] unaware of itself as it moves through the world.

Our teaching of mathematics is founded, at least in part, on the premise that formulated understandings are in some way better than those which remain unformulated. This is not a difficult point to argue; one need only call attention to the tremendous organizing and predictive "powers" of certain concepts. However, it is a premise that becomes troublesome when the state-able is privileged over—or considered in ignorance of—the enacted. Rather than focusing strictly on the promotion of abstracted (and therefore formulated) understandings, it might be more appropriate if mathematics educators were to also look at the interplay of formulated and unformulated understandings. How do they affect—that is, support, negate, or shape—one another?

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[26] Vygotsky, *Mind in Society: The Development of Higher Psychological Processes*, 26.

[27] Johnson, *The Body in the Mind: The Bodily Basis of Meaning, Imagination, and Reason*.

[28] This idea has been developed by Wilhelm Reich. See Morris Berman, *The Reenchantment of the World* (Ithaca, NY: Cornell University Press, 1981).

Important inroads into this issue have been made by a number of theorists and researchers in mathematics education. Worthy of note are those constructivists[29] who contend that the learner's actions, and not socially-sanctioned pre-established truths, are the most important source of one's mathematical understandings. Regarding the interplay of these informal actions and formal conceptualizations, Thomas Kieren and Susan Pirie[30] have developed a model of mathematical understanding that serves to bring out the complex back-and-forth movement between immediate and interpreted experience. Their model, which they illustrate with a series of concurrent circles (see Figure 4), is intended to portray the *non-linear* movement from one's current knowledge (in their terms, "Primitive Knowings") through to more formalized understandings. The various levels of mathematizing behavior represented in the different circles are arranged in order of increasing abstraction. They are not intended to represent a series of incremental steps one must take on the path to formal understanding, but the range of possible ways of acting mathematically. As such, one might skip several layers in a flash of insight, or one might be compelled to "fold back" to account for an unexpected result.

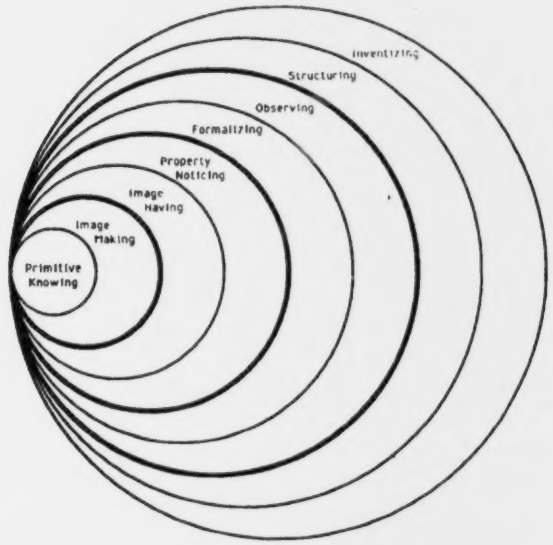

Figure 4. The Kieren-Pirie Model of Mathematical Understanding[31]

The Kieren-Pirie model of mathematical understanding is not intended to serve as a basis of predicting mathematical behavior. Quite the contrary, it is predicated on the belief that such behavior is complex and unpredictable. It is thus a model to enable the observer or listener's description of what has happened—what the learner has done and the invocative or provocative (i.e., causing the learner to "fold back" or to skip ahead) consequences of a teaching intervention. Its important contribution, for the current purposes, is the formulation of different modes of acting, replacing the rather unhelpful and rigidly dichotomous concrete-abstract distinction. Understanding, as their model and

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29 Again I emphasize that the term "constructivism" is being used strictly in reference to "radical constructivism.

30 Pirie and Kieren, "Growth in mathematical understanding: How can we characterize it?"

31 Taken from ibid. Reprinted with permission.

their supporting research indicates, is a dynamic and active process of negotiating and re-negotiating one's world whereby the abstract can never be severed from the concrete.

Each level of the Kieren-Pirie model is associated with particular styles of logical reasoning, and so a researcher using the model must be aware of the sorts of actions and expressions that might be considered proper to each level. For the listening teacher, this is an important aspect of the model, for the movement from one level to another might be (and often is) occasioned by questions or other teaching actions. The teacher must thus be able to listen for the characteristic logics.

Further, as regards the distinction that I have been drawing between formulated and unformulated knowings, Kieren and Pirie have developed the idea of "don't need boundaries" (indicated by the darker circles in the diagram) to highlight particular events in the growth of one's dynamic understandings. As one crosses a "don't need boundary," one is able to operate at a more sophisticated level without having to rely on previous modes of thinking and acting. For example, a learner whose understanding of fraction addition has been tied to his actions in manipulating materials might begin to talk about the additive process in terms of those materials but without having to actually manipulate them. Later still, he might begin to formally operate on mathematical symbols without having to refer to the original manipulative activities. In each instance, the learner has moved to a level of understanding in which previous actions or processes, while still available should he need to fold back, are not explicit parts of the current understandings. In terms of formulated and unformulated knowings, we observe in such examples the manner in which each unfolds from and folds into the other. What was once formulated (e.g., manipulating fraction pieces) comes to be taken-for-granted (unformulated), but still remains available for interrogation.

Formulating one's understandings is regarded as a process analogous to *storying*, whereby the learner incorporates new experiences into the text of previous understandings. In this process, both past knowings and interpretations of current experiences are revised—one rewriting the other, as it were. This revisionary and recursive *storying* process stands in stark contrast to the cumulative and linear *storing* process of transmission-acquisition models of learning (especially those founded on "mind as computer" metaphors), thus highlighting not just the *reproductive* nature of understanding but its *productive* aspects as well. Understanding is generative—both of the selves that participate in the understanding and of the world that is opened (or "brought forth") by that understanding.

The Kieren-Pirie model might thus be interpreted as a framework for listening, for it places the teacher/researcher in a necessarily attentive relationship to the learner. The model demands not just that one be more mindful of student articulations and actions, but, because such activities are argued to be of benefit both to teacher and to learner, one must present occasions for learners to express themselves verbally and in action. It is thus that this model acknowledges the dialogical structure of our mathematical knowings. The teacher is seen as part of the action and is hence implicated in all the emerging understandings.

Meanings of Meaning

A distinction between a *meaning* and an *understanding* is not easily drawn—and our inability to construct one is perhaps attributable to the ranges of understandings of "meaning" and meanings of "understanding" that are present in our day-to-day language.

In many cases, the words are used interchangeably; in others, they are separated through rigid definition.

Here I am considering *meaning* apart from *understanding*, not because I believe them to be in any way separable, but because the word "meaning" has taken on a particular status within the field of mathematics education. There the meanings of "meaning," for the most part, demonstrate the same tendencies to privilege the formulated and to neglect the bodily as the most popular understandings of "understanding."

Conventionally, *meaning* tends to be understood in terms of connective associations. On the more objectivist side, the *meaning* of a concept has to do with its relationships to other concepts, its possible applications, its derivation, etc. In short, *meaning* is something that can be identified and subsequently taught. More subjectively, *meaning* is understood in terms of personal associations to other ideas, and it is a quality to be either supported or contradicted, depending on its appropriateness. In both cases, *meaning* is understood in formalized and formulated terms: it is something that can be, and usually is, *stated*. As such, *meanings* reside in the domain of language, and language, in turn, is generally cast as a mental (in contrast to a physical) capacity.[32]

To the enactivist, however, the bulk of our *meanings* are neither formulated nor strictly linguistic. They are, rather, lived through or enacted. For the purposes of illustration, we can turn to Hilary Putnam[33] who describes two opposing conceptions of the *meanings* of words. In formal terms, to know a word's meaning is to be able to provide a definition without using the word itself. However, as Putnam points out, most of the time we are unable to readily offer such definitions, and yet we clearly *know the meaning* of the words we are using—simply by virtue of the fact that we are using them. (Conversely, one's ability to provide a formulated definition is no assurance that the word can be used meaningfully.) Their meanings are thus enacted, and it is to this unseen (or unacknowledged) part of the iceberg that enactivist theorists invite us to focus our attentions in our discussions of education. The measure of learning is not what can be stated or re-stated, but what is performed. Put differently, thought and action are not precursors to or consequences of *meaning*; they are *meaning*.

In a similar vein, Richard Rorty refers to such *enacted meanings* (i.e., our languaging behaviors) as "patterns of acting," a phrase that underscores the notion that our language is not a "third thing" that exists between you and me. Rather, our language (i.e., our speaking, our conversing, etc.) is an element of our moment-to-moment acting and interacting. Applying Putnam's and Rorty's notions to a discussion of the meaning of mathematical terms, symbols, and concepts is not difficult. We need not delve too deeply into our own ways of interacting with the world to find that we are continuously enacting particular meanings or understandings of mathematical ideas—we compare, we note patterns, we prove, we group, we abstract, we deduce—all in ways that are particular to a culture in which mathematical reasoning is privileged.

Here our enacted meanings are embodied meanings. Arising from the background of unformulated experience, passing through some manner of formulation, these meanings return to silently shape the actions of the body. They become part of our being—part of the way we stand in the world. They become our "common sense," our

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32 Not unlike the orientations to cognition that were introduced and critiqued in the first section of this chapter, such orientations to meaning (and to language) might be called "representationist." Words are thought to stand in for objects and concepts, and hence must themselves be considered as objects. The alternative offered by enactivism is that words are part of our complex patterns of interactive behavior.

33 Hilary Putnam, *Representation and Reality* (Cambridge, MA: The MIT Press, 1989).

taken-for-granted, the "way things are," sometimes so blindingly obvious that they resist re-formulation and re-presentation. There is thus a dialectic between formulated and unformulated meanings. Learning is a process of affecting one's meanings—formulated and unformulated, for the two cannot be considered apart. It is a re-structuring of being. The key element is that what is learned must be in some way meaningful.

Herein lies the problem with conventional approaches to mathematical instruction that begin by presenting a concept and proceed by attempting to render it meaningful. We are, in effect, saying, "Here is something that is not about you; here is how you make it about something that is about you." Sometimes this approach works. Most of the time—as evidenced by popular (Realist) belief as to the nature of mathematical knowledge—it holds knower apart from known. With regard to mathematics education, this separation is a tragic one, for the bases of almost all of the meanings and understandings that we seek to promote in grade school are found in everyday—albeit unformulated—experience.

Perfinking

Although I'm treating the terms as more-or-less synonymous, there is perhaps an affective element suggested by "meaning" that is not associated so strongly with "understanding." People, events, objects, memories can become *meaningful* for reasons that we very often are unable to comprehend.

This facet of *meaning* has been virtually unaddressed in mathematics education, where thought and emotion are separated in yet another modernist dichotomy—a split that is supported by the pervasive opinion that the discipline is objectively neutral. Any personal or emotional associations to the subject matter are, in consequence, thought inappropriate and irrelevant. Indeed, the word "rational"—used as it is to describe both mathematical modes of reasoning and sane, sensible, unaffected behavior—points to a sharp distinction between reasoned thought and *irrational* feeling. In the modern frame, thought is considered to be of the mind, and feeling of the more primitive, animalistic body; and so the thought-emotion split might be considered another relic of our Cartesian heritage.

As Walkerdine[34] persuasively argues, the disdain for the emotional—and the belief that the rational might provide us with a means of overcoming our irrational proclivities—has exacted a toll from learners of mathematics. In the classroom, where a sometimes "complex" and "painful" suppression of experiential knowings is required to attain a mechanical proficiency, learners are stripped of a dimension of their being.[35] Often the ignored feelings or suppressed emotions are manifested in a hatred for the subject or an indifference toward the teacher. Other times it creeps in more subtly in the throw-away remarks that students so often make: "My family's bank account is in the negative integers," "With my mother gone, only four fifths of my family remains." We underestimate, for the most part, the emotive powers of our numbers, forgetting the way they permeate our existence.

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[34] Walkerdine, *The Mastery of Reason: Cognitive Development and the Production of Rationality.*

[35] I do not mean to overstate the case here. Not everyone "suffers" in learning. There are those—myself included—whose experiences with mathematics are far more "friendly" and for whom the learning of mathematics is clarifying rather than obfuscating. The extreme example of this phenomena might be those autistic persons who find comfort and respite from the incomprehensibility of human emotion in the reductive and consistent predictability of the mathematical. Oliver Sacks, in "A Neurologist's Notebook: An Anthropologist on Mars," presents a case study of one such person.

In many ways, the severance of thought from feeling is an odd one, especially for those of us who, like young children, have retained the capacity to be excited about what we do and do not know. As Margaret Donaldson puts it, "passionate curiosity empowers the intellect. . . . [The] achievement of new understanding is normally accompanied by delight."[36] There is a devotion that animates the existence of those who seek after new truths—a passion that is very often absent in both the teachers and the students of conventional school mathematics. The evidence of this statement is found in the school definitions of "discipline," a word that has lost its original sense of devoted following to become associated with imposition, regulation, and punishment.

David Krech[37] has coined the term "perfink" to draw attention to the way we perceive, feel, and think at once. Bruner, elaborating on Krech, suggests that action is an important element of our perfinking. All of these elements are present as we learn—as we interactively make sense of our world. That only rational thought has been the domain of mathematics teaching, then, has been its most dehumanizing trait—one that has been supported by a uni-directional teaching style (teacher-to-learner) that has seen no value in attending.

The perfinking teacher, however, cannot help but listen.

The Space of Learning

The Western notion of Self—that thing which thinks, understands, means, feels, and acts—is inadequate for the purposes of understanding learning and teaching, primarily because it obscures the transformative process that is the self.

I return to this issue to add a note of elaboration, for even with a more fluid definition of identity, we still tend toward thinking of our selves as agents that "have" or that "shape" understandings, meanings, and feelings. While I am uncomfortable in suggesting that this notion is without value, I do think it lacks a necessary symmetry. Our understandings, meanings, beliefs, feelings and actions, in other words, are not so much things that we give shape to, but events that give shape to our selves. As Rorty puts it, "Just as the brain is not something that 'has' . . . synapses, but is simply the agglomeration of them, so the self is not something which 'has' the beliefs and desires, but is simply the network of such beliefs and desires."[38]

This point is an important one because, in many ways, learning involves an attempt to resolve the tensions that arise between tacit and explicit knowledge, between emotional and reasoned actions, between intuitive and calculated responses. At times these move in harmony, at times they exist in contradiction. In colloquial terms, the issue here is the level of agreement between "what we say and what we do." Unfortunately, this phrase tends to be expressed derisively—as a statement of bad faith. We are critical of someone who says one thing and does another.

These inconsistencies of our behavior might more profitably be regarded as small tears in the fabric of our existences—ones that offer a hand grip or a toe hold for movement. Lacking them, there is little need to learn. And so, putting it into more formal terms, there is always *play* between the formulated (state-ed) and the unformulated

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[36] Donaldson, *Human Minds: An Exploration*, 141.
[37] Cited in Bruner, *Actual Minds, Possible Worlds*.
[38] Rorty, *Objectivity, Relativism, and Truth: Philosophical Papers, Volume 1*, 123.

(enacted)—where the latter provides evidence for or contradicts the former; where the former validates or reframes the latter. It is in this *play*—the movement, the divergence, the synchrony—that we find a space for learning.

An illustration. In the middle of a unit on decimal numbers, Melinda, a grade seven student, asked her teacher for assistance with the question: "Calculate $3 \times \frac{1}{4}$. Express your answer as a decimal." The teacher's first attempt at assisting was to ask, "If you had three quarters, how much money would you have?" Without hesitation, but with a decidedly quizzical look, Melinda responded, "Seventy-five cents."

She saw no connection between the original question and the one posed by the teacher—between the formal mathematical concept and her enacted mathematical competency. As she and the teacher explored the play between the two, however, Melinda's understanding of both was broadened.

In the last section of this chapter, I delve more deeply into the place of this sort of play in the learning of mathematics. My orienting theme is that we, as educators, need both to open spaces for and to locate existing instances of tensions between stated and enacted meanings, understandings, and knowings.

Section C

Play

Play—a sacred activity which,
in our see-and-work world,
has been made profane.
— David Denton[39]

"Play" is really quite a wonderful word. If I might be permitted for a moment to think of it as a box filled with meanings and possibilities, then *play* is the sort of container that lures the person intent on play. It invites itself. It announces itself. It enables itself.

As I examine its many dictionary definitions, it seems that I am faced with a choice. I might treat it as a word with numerous and divergent applications—used in the contexts of sport, frolic, drama, jest, love, and risk, to name a few—or I might play with it as an essentially human quality that is hinted at, but never fully revealed in, each of its definitions.

In this section, I try to do the latter, but precede the discussion by acknowledging the futility of attempting to capture and tame this play-full thing-no-thing. It cannot be found or created, it cannot be planned or manipulated. It exists in the immediate. Play is only play in the playing. And so, I play.

Education and Play

In English, the link between education and play is not apparent. Not surprisingly, then, in the conventional school, play is discounted as childish, haphazard, ineffective, and inefficient. Among the subject areas, mathematics classes are perhaps the least playful. There, even the words governing what is to be learned have been stripped of their play: they are rigid and restrictive, defined in ways that belie their sensual origins and discount their associated experiences.

In Greek, however, the distinction between play (*paidiá*) and education (*paideía*) is not so obscured. Both terms arise from an original reference to the activity of the child (*pais*), an echo of which can be heard in the word "pedagogy" (*paidagogos*). Plato incorporated this play-element in his *paideía* as he recommended that the guards' children learn their lessons through *paidiá*. But, even for Plato, the connection between play and education was quickly lost as he moved to the "higher form" of his dialectic. There learning became a matter of earnestness.

Piaget regarded play as the most powerful of the child's learning activities. In play, the child takes what surrounds him and, in effect, re-structures that which was given into a range of imaginative possibilities. His physical action and his "make believe" enable an internalization of certain parts of the world, leading to a broader range of possible actions. Play, in Piaget's conception, is thus a creative endeavor which is understood in the child's orientation to the activity rather than in the activity itself.

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39 Denton, *Gaia's Drum: Ancient Voices and Our Children's Future*, 7.

However, with Piaget's conception of play, the educative import is lost as one progresses to more complex modes of thinking and reasoning. In contrast, John Dewey articulated a perspective on play that can be applied across developmental stages and maturational levels:

> The first stage of contact with any new material, at whatever age of maturity, must inevitably be of the trial and error sort. An individual must actually try, *in play*, to do something with material . . . and then note the interaction of his energy and that of the material employed. This is what happens when a child at first begins to build with blocks, and it is equally what happens when a scientific man in his laboratory begins to experiment with unfamiliar objects.[40]

Unfortunately, the perspective on play in the secondary mathematics classroom is neither that of Piaget nor that of Dewey, having been made synonymous with "off-task behavior" and "goofing around." Its re-creative aspects have beef forgotten as it has become associated strictly with recreation; play is now thought of as something that interferes with the serious business of schooling—something to be purged, held down, put off. But the playfulness of learners is not so easily suppressed.

An illustration. Early one school year I asked on a mathematics quiz, "What is the difference between 18 and 8?"—a poorly worded question intended to help me determine who among a class of grade eights knew the mathematical definition of "difference." Perhaps one third of the students gave the "correct" answer of "ten." The rest suggested that the difference had to do with the fact that 8 was smaller, less, lower, or that it had one fewer digits; or that 18 had more factors, or that it had a number in the "tens place." One learner provocatively suggested, unfortunately without explanation, that there really was no difference. All these responses I marked wrong. There was no play in my grading. There was no play in my understanding. Unable to suppress the play-fullness in the learners ideas, I was compelled to ignore it and to punish them. My inability to listen led me to close the gate on a mathematical playground, rich in its potential.

So far in this section, I am been deliberately using "play" in many different senses. My attempt at a playful understanding of the word has perhaps run its course, and so I return to the serious business of explication. As I move through the remainder of this section, I draw primarily on the work of Hans-Georg Gadamer to come to a deeper understanding of play and, in the process, to point at the centrality of its place in the mathematics classroom. Play, like listening, is a phenomenon that I argue has been shallowly understood and, in consequence, almost universally scorned by mathematics teachers. Motivating this discussion is my belief that a deeper understanding of the role of play in learning will not only provide cause to embrace it as an aspect of one's pedagogical practice, it will highlight the place of listening in the lives of mathematics teachers.

I must note at the outset that my use of the term "play" continues to be deliberately ambiguous. My refusal to assign it a rigid definition—to "pin down" what I am talking about—springs from my firm conviction that there is much more to be learned by playing with the possibilities that by mechanically reducing them to a single definition. That is to say, I am purposefully challenging the modern dichotomy that holds apart the irrational, imaginative, bodily, sensual, and fictive realm of play from the rational, mathematical, mental, austere, and fact-ive realm of reason.

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[40] John Dewey, *Democracy and Education* (New York: The Free Press, 1966 [1916]), 180-181. (emphasis added)

The Essence of Play

Exploring the meanings of *play* in such phrases as "the play of light," "the play of the waves" and "the play of forces," Gadamer suggests that the essence of play is "movement as such" which "has no goal that brings it to an end; rather, it renews itself in constant repetition."[41] It is thus that he uses the concept of play as he attempts to decenter the notion of subjectivity, for this movement is impersonal and not subject to subjective control. Rather, within play, subjectivity loses itself; at some point, the game takes over. In retrospect we say, "I forgot the time," "I don't know how I did that." There remains, however, a subject of play, but that subject is *the play itself*. The game takes primacy over the players, just as the subject of play takes primacy over the subjective consciousness of individual players. In Gadamer's words, "all playing is a being played. The attraction of a game . . . consists precisely in the fact that the game masters the players."[42]

It is important to note that, for Gadamer, play is not the opposite of seriousness. Quite the contrary, "seriousness in playing is necessary to make play wholly play."[43] This notion recalls a pre-modern usage of the term by which, according to David Denton, "to play meant to pledge, to stake one's life, to guarantee, and it also included something of its opposite; that is, the dangers and perils of such mutual commitment."[44] The relevance of this point becomes clearer as Gadamer moves to a discussion of the role of play in one's experience of a work of art. As mentioned in the preceding chapter, a work of art has the capacity, while representing, to present something new and thus to challenge the taken-for-granted-ness of daily existence. Gadamer explains that artwork has this capacity because it is a form of play. It exists in the play of what is *known* to be true and what might otherwise be thought. It invites play. It draws the player out of himself and, in the interplay, opens a space for transformation. Subjectivity loses itself and re-emerges having learned something—having been changed—and so the playful loss of subjectivity is instructive rather than destructive.

It is thus that play educates. It presents the as-yet unexperienced, the unpredictable, the uncontrollable. As Gallagher puts it:

> The unfamiliar that we experience in play is first of all interpreted in terms of the world. In play we become so fascinated with the world that we move beyond ourselves, we transcend the limits of the self. . . . The self-transcendence that is essential to play involves a projection toward one's own possibilities. . . . *Play bestows reality on the unreal; it gives weight to that which is possible or fanciful.*[45]

It is here that the moral significance of play is revealed, for if we accept the earlier conclusion that the self is a process of change, then play is a catalyst for this transformation. Indeed, as we speak of education, it may even make sense to suggest that the movement of play is an essential element of all educational experience.

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[41] Gadamer, *Truth and Method*, 102. I might add here that the "repetition" of play should not be confused with the repetitiveness of certain activities—such as rote recitation or drillwork. Rather, within the repetition itself, there is movement (play), so that each act of repetition is indeed a new (informed and transformed) act. It is thus that play sustains itself.

[42] Ibid., 106.

[43] Ibid., 102.

[44] Denton, *Gaia's Drum: Ancient Voices and Our Children's Future*, 158.

[45] Gallagher, *Hermeneutics and Education*, 50. (emphasis added)

"Just playing around"

Friday was always "Problem Day" when I taught. Students would be divided into groups and assigned three or four "non-standard" (for that was my definition of "problematic") questions from the text.

Most of the time the students dutifully, but somewhat dispassionately, worked at the assigned task. From time to time, however, a question would capture the interest of one student or another, and on some occasions, that interest grew to a contagious enthusiasm. I could usually tell when we were on the verge of one of these events. The sign was a change in some student's response to my standard intrusive inquiry, "What are you doing?" The normal answers were along the lines of, "We're on number seven," "Thinking," or "Getting back to work." But occasionally someone would say, "Oh nothing, we're just playing around with this question."

Early in my career I interpreted this response as an indication that the students were "stuck." They were trying some things, but could make no progress. More often than not, I immediately ignored what they were doing and undertook to explain the problem into mundane-ness, enabling them (I thought) to get back to the serious business of mathematics. Later I found that by asking them to explain what they were "just playing around" with, interesting things were happening. More recently, I have had occasion to watch over the shoulders of students engaged in mathematical tasks. It often happens that I am unable to make sense of what they are doing; I can describe it only as "play." And, not surprisingly (any more), they describe it in the same way when asked. "Just playing around."

The key element in these cases is not the making of progress, but the allowing of space for movement. My inability to notice the importance my students' playing was the "aroundness" of event; it seemed incompatible with the linearity of the curriculum I sought to follow. "Playing around" suggests a turning and re-turning, a back-and-forth, a repetition and recursivity that are perhaps more in harmony with the ways we learn and live that the lock-step, straight-forward structures of many textbooks. "Around," like "play," is a polysemous, playful word that we use to allow ourselves space for movement.

The "just" part of "just playing around" is also important. It indicates two things to me as a teacher. First, playing is something we tend toward. When work is done (or forgotten), when duties are fulfilled, then we can *just* play. It goes along with just thinking, just resting, just fooling around. Second, and perhaps more importantly, it hints at a somewhat disparaging attitude toward playing. Such attitudes are to be expected for the conventional mathematics classroom, as mentioned, is not a place for play. Mathematics learning is conceived to be about the *mind*: about state-ing understandings, about fixing meanings. Play, in this conception, points to the "opposites" of these modern ideals: to loosening and fluidity; to the *body* and to acting. But are these true opposites?

Gadamer's explication of the centrality of seriousness in play would suggest that they are not. The formulated and the stated are critical elements of the unformulated and the enacted. They are the markers that reveal the hidden movement. The problem, then, is not so much that we have focused on the serious, the formulated, and the fixed, as that we have set them up in contradistinction to play. The modernist quest for progress is to blame here, for in our desire to move toward pre-specified goals, we have become disdainful of "just" moving . . . *just playing*. In focusing on the rules of mathematics, on the limits of the individual players, on the bounds of the playing field, on the time clock, we have lost touch with the play.

Play Time

Perhaps the most devastating phenomenon associated with this loss is the attitude toward time that goes along with mathematics classes. There, time is a commodity. Some researchers,[46] in fact, have gone so far as to prescribe a regimented structure for mathematics lesson that they contend makes the most efficient use of time and produces the most effective results. The de-humanizing metaphors underlying this sort of project are clear. Yet, for the most part, they go uncritiqued. Mathematics learning, after all, is important business.

The criterion of efficiency and the demands of planning compel the teacher to hold a particular perspective on time: a resource that is limited and precious. In Alan Lightman's words, this time is mechanical, predetermined, and unyielding; it is "as rigid and metallic as a massive pendulum of iron that swings back and forth, back and forth."[47] Homework should thus be checked in the first seven minutes, the lecture (with examples) completed in the next ten, and the remaining time fragmented among the other essential lesson elements. If an interesting question arises, one checks one's watch to determine if there is time to pursue it. If a student wastes time at play, it is recorded and "made up" at lunch or after school. This mechanical time orders life, it does not obey it.

The time of play is different. It is a "body time" that "makes up its mind as it goes along."[48] It is a time that moves in fits and starts to the rhythms of moods, desires, heartbeats. It is the time that races between the bells that mark the breaks and that slows between the bells that bound classes. It is time that can be frozen by an icy glare, or that can fall away when absorbed in conversation. It is a time that obeys the body.

That is, it *listens* to the body.[49] In our world, and especially in our schools, it is a forgotten time, for in the classroom the body is made to submit to the order of mechanical time. "Put that food away. It's not time for lunch yet," "No, you can't go to the washroom. Why do you think we give you a break between classes?" And, most commonly, "Get back to work. The time for play is after school."

Mechanical time is the time of *telling*. It is the time of rote, recitation, regurgitation—of *dis*-play. Body time is the time of *listening*. It is a time of pause, passion, persons—of play. It is a time that takes the time that is needed. It is a time that does not move in lock step, that does not parse knowledge into 45-minute gulps, that does not hastily scribe an "X" beside the different answer. It is a time that spills from class into break, from school into home, from one day into the next, from one topic into another.

"Where the two times go their separate ways, contentment." Where the two times meet, desperation."[50] They meet in the modern mathematics classroom. The violence of their encounter, however, need not be so great. This is where recasting teaching as listening enters, for listening involves a forgetting of modern linear time. The listener is not subjected by the object of time. It cannot be an issue.

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[46] T. L. Good, D. A. Grouws, and H. Ebmeier, *Active Mathematics Teaching* (New York: Longman Inc., 1983).
[47] Alan Lightman, *Einstein's Dreams* (Toronto, ON: Alfred A. Knopf Canada, 1993), 23.
[48] Ibid., 24.
[49] Etymologically, "to obey" (from the last line of the preceding paragraph) mean "to listen from below."
[50] Ibid., 27

Conversations: Word Play

A conversation can be put to a quick death with a simple glance at a watch. This tiny act yanks us back into mechanical time where we must be getting places, where we must be getting things done. Another way to kill conversation is to arrive at some sort of terminal (state-ed) "understanding" that neither requires nor allows for further movement. These sorts of constraints are the rule of the conventional classroom. As far as teaching goes, it is they that make it unnecessary and impossible to really listen, not just because listening happens in body time, but because listening and conversation are sorts of play.

Words *play with us* as they move, twist, and disappear. They *play us* as they tug, push, and taunt. This play is perhaps most clearly evident in a conversation in which, through words, we are conducted through the play of meanings. It is thus that Gadamer notes that play and the conversation have the same hermeneutical structure. If the play (the give-and-take in meaning) of a word could somehow be overcome, there would no longer be any need to listen: either there would be no room for misunderstanding, or there would be no possibility of verbal communication. Listening, then, and *not* speaking, is the human capacity that enables interaction. It is that capacity that makes it possible to make sense of and to maintain sanity amid a sea of linguistic messiness, ambiguity, and play. In David Denton's words, "listening is full participation with that being heard, which surrounds like the water and sun, in depth, multisensually, with no sensation of time."[51]

It is also the play of words that frustrated Descartes' Rationalist project. His goal was to construct a positive and certain philosophy, free of presuppositions and guided by the clarity and rigor of mathematics.[52] He was compelled to use words, however, which, in spite of his efforts to legislate their meanings, could not be detached from the traditions that defined them. Nor could their ambiguity be squeezed away. Put differently, the words had too much play to serve as the building blocks of an unshakable philosophy. It is for precisely this reason that postmodern critics have focused their analyses on language. Their project is founded on the notion that our language, while enabling us to render sensible the meaningless and the chaotic, can always be turned destructively upon itself. Nothing is certain.

Play School

And so *play* is much more than childish or imaginative activity. It is an essential human quality. The argument that play must figure more prominently into the learning and teaching of mathematics is therefore *not* a call for greater activity in a program already packed with too much busyness. Nor should it be interpreted to suggest either that interactive participation is preferable to individual work or that attempts at structure and routinization are ill-advised. On the contrary, implicit in a deepened understanding of *play* is the notion that it can be realized (and is sometimes *best* realized) in stillness, in solitude, or in repetition. Put simply, play is not so much an activity as it is an acceptance of uncertainty and a willingness to move. Play is thus the antithesis of the modern ideals of certainty, predictability, and linear progress. But it is not an abandonment of our quest

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[51] Denton, *Gaia's Drum: Ancient Voices and Our Children's Future*, 114.

[52] In Chapter 2 I argued that Descartes' project has largely been realized. I continue to maintain that position, in that mathematics has achieved a pervasive presence in almost all aspects of modern life. The point being made in this chapter is that, in spite of this "success," the Rationalist project of Descartes, from the outset, was destined to failure. It bore the seeds of its own destruction.

for structure, order, pattern, and comprehensibility. Quite the opposite, these are the goals of play.

But these goals are only revealed in the playing, for play is not simply random activity. Rather, by opening the door to the as-yet unexperienced, to the possible, play reveals what is not yet known as it simultaneously offers space to support learning. (And, importantly, this is a space of enaction rather than inaction.)

The manner in which this contiguity of play and learning has manifested itself in the modern classroom is somewhat disconcerting at times. As William Pinar, in a critique of current conceptions of curriculum, poignantly puts it:

> In schools, particularly in secondary ones, and those for higher learning, one observes countless persons playing at being a student, a professor, an intellectual, a radical, a bohemian, a freak, and so on, playing at being some thing other than themselves. They are not themselves; in Laingian terms, they are out of their minds; they are mad.[53]

There is play; there is learning. But what are the games and the gains? Pinar is arguing, in effect, that the structure of the conventional school compels learners and teachers alike to assume pre-defined roles, to move along pre-specified paths—to play a de-humanizing game—rather than to participate as the persons they are (becoming). It is thus that, in teacher education, we speak of such processes as "developing a teaching identity," inadvertently bringing to being fragmented teaching selves, social selves, learning selves—"playing at being something other than [our]selves." But there are alternatives to playing these roles—alternatives that embrace rather than suppress the play-fullness of learning.

In the classroom, then, the recognition of the vitality of the connection between play and learning points to a participatory sort of teaching—a teaching where the teacher does not stand outside to direct the play, but becomes a vital part of the action. Immersed in the play, the teacher too is a learner. As van Manen explains

> Etymologically, to learn means to follow the traces, tracks or footprints of one who has gone before. In this sense, the teacher . . . who is able to "let learn" therefore must be an even better learner than the child who is being "let learn."[54]

And so the teacher is a learner with particular responsibilities. He is assigned the tasks of presenting possibilities and, through attending to the student's responses to these possibilities, opening spaces for play. Such play-fullness is only feasible when one allows for departure from the anticipated (play), fluidity in the structured (play), and uncertainty in the known (play).

Here is the place of listening—of being part of the rhythm and movement of the classroom, of living (rather than merely fulfilling) one's teaching role.

I end this chapter with an illustration. This one I set up as a contrast. First I recount a lesson on fraction multiplication as I taught and as I was taught. Then I present a piece of a lesson on fractions where rigidity is melted to play and teaching is enacted through listening instead of telling. These accounts are offered without immediate

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[53] Pinar and Grumet, *Toward a Poor Curriculum*, 11.

[54] van Manen, *The Tone of Teaching*, 44.

commentary, for they are intended not just to illustrate the place of play, but to point at what enactivist teaching might be. That is the topic of my final chapter.

Please listen for the contrasting senses of seriousness, time, structure, and understanding in these accounts.

Frac(tured)tions—No Play

As the bell rings, Wendy moves to the front of the room. Above the students' voices we hear hers: "ONE . . . TWO . . ." The class falls silent before she reaches "three." Wendy waits a few more seconds, her eyes scanning the room until everyone is properly seated in their desks.

"Take out your homework and pass it to the person in front of you." There follow five minutes of calling out answers, moving from student to student, up and down the straight rows. The exercises are corrected, the scores are tallied, the numbers are called out and recorded.

"Okay, we're moving on to multiplying fractions today. Before we get started, I want to make it clear that the way you multiply fractions is totally different from the way you add fractions. I don't want you to confuse the two."

A ten-minute lesson ensues in which the multiplication algorithm is presented along with a variety of examples that cover the anticipated contingencies. As the lesson moves along, students are given a few "Do Now" exercises and, after a moment of hasty computation, are asked to report their answers. Notes are written on the board and copied into workbooks. The students ask few questions.

The assignment—a textbook exercise—is then announced and the students are given the balance of the class for independent work. During this time, Wendy moves from raised hand to raised hand—confirming answers, pointing our errors, and repeating fragments of earlier explanations. A reminder that the "rest of the page" is to be completed for homework is given as the bell rings to end the class.

Frac(tal)tions—Play-Full

Tom wanders among his grade seven students as they play with their *Fraction Kits*,[55] sheets of paper cut into halves, thirds, and other fractional pieces. He has posed a question to the class: "What can you say about three fourths?"

The class members are noisy and active. Groups are huddled together, most of them using the kits to devise methods of covering three fourths of a piece of paper. As Tom moves about the room, he listens, he questions, he requests that students display their work on the chalkboard.

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[55] *Fraction Kits* were designed by Thomas E. Kieren. A fuller account of the materials and the sorts of activities they support is provided in Thomas E. Kieren, Brent A. Davis, and Ralph T. Mason, "Fraction Flags: Learning from Children to Help Children Learn," in *Mathematics Teaching in the Middle School* (forthcoming).

Not everyone is finished when he calls them to a discussion of what they've done. Interesting insights and possible avenues for further investigation are presented. Sarah asks a question: "How many different ways can you make three fourths using the pieces from the kits?"

"I don't know. I'm not sure if anyone knows," is Tom's response. "I wonder if we can figure that out."

Activity resumes as the groups produce five, six, nine, eleven, answers. As Tom listens in on one group, he notices that Greg is using the charts differently than his classmates: as a generative tool rather than merely as a recording device. With it he can determine ALL the possibilities for three fourths—and for any other fraction quantity—quickly and efficiently.

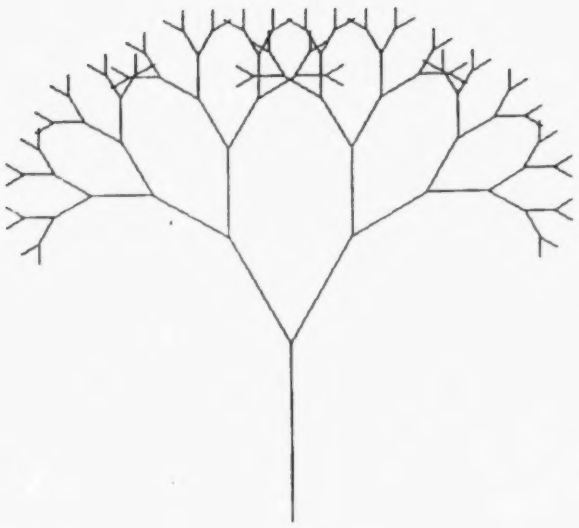
Noting that Jake, Greg's partner, is copying Greg's responses without really participating, Tom asks Greg to explain his reasoning. Greg complies, but it is clear that Jake doesn't understand. However, the event does prompt him to stop copying and to return to manipulating the pieces of the kit.

Moments later, Tom feels a small poke in this back. He turns to see Jake standing there, holding up his chart. On it an interesting pattern has been recorded.

"I know everything about '1'," Jake announces.

Chapter 5

PLAYING IT BY EAR
Teaching



Many violinists and violinmakers insist that violins grow into their beautiful throaty sounds, and that a violin played exquisitely for a long time eventually contains the exquisite sounds within itself. Somehow the wood keeps track of the robust lyrical flights. In down-to-earth terms: Certain vibrations made over and over for years, along with all the normal processes of aging, could make microscopic changes in the wood; we perceive those cellular changes as enriched tone. In poetic terms: The wood remembers. Thus, part of a master violinist's duties is to educate a violin for future generations.

—Diane Ackerman[1]

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[1] Diane Ackerman, *A Natural History of the Senses* (New York: Vintage Books, 1990), 204.

Section A

The Nature of Teaching

> My own view is that it is not much use pointing to the "internal contradictions" of a social practice, or "deconstructing" it, unless one can come up with an alternative practice.
> — Richard Rorty[2]

No where is the "theory-practice" tension more visible than in the field of mathematics education.

If I have been correct in my evaluation of the forms that modernist dyadic perspectives have assumed, then this announcement should hardly come as news. "Theory versus practice" is just one more Cartesian dualism to add to the list already assembled.

If I might be permitted to draw a fairly vulgar line between the two "camps" within mathematics education, at the moment we have the theorists and researchers on one side, gathered under the banner of "constructivism," and the teachers (practitioners) under siege on the other, living out a realist perspective within the increasingly unsafe bounds of the classroom walls. That impassable line between them is marked by the corpses of the many ill-fated efforts to define the term "constructivist teaching."

My purpose in this section is to examine this impasse more closely. First I ask, Why is it that theorists and teachers seem unable to communicate on this issue? And then, How might we go about overcoming or circumventing this most devastating tension?

The State of the Dis-Union

As I have elaborated in the preceding chapters, the two faces of modernism have created a sort of crisis of identity for mathematics educators, torn as they are between Realist/Empiricist and Rationalist/Constructivist perspectives. The former (whereby knowledge is an object, teaching is the transmission of knowledge, and learning is a process of acquisition) underlies the entrenched orientations to and methods for teaching. Backed by current institutional constraints, program demands, public expectations, and the momentum of tradition, the realist perspective militates against any sort of radical reformation. The latter (whereby knowledge is fallible, education is enabling, and learning is a process of constructing) serves as the theoretical foundation of an extensive body of research where only a few—if any—current reports fail to acknowledge an allegiance to the constructivist orientation.

The realist view of instruction, founded as it is on a "conduit" model of communication, might be succinctly described as a process of *telling*. So conceived, the teacher's task involves first selecting the bits of knowledge to be passed on and then re-presenting them with a mechanical efficiency. The emphases here are on clarity of expression, depth of explanation, structure of practice, and dissection of knowledge (into appropriately-sized transmittable bits). Madeline Hunter, a prominent proponent of this

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[2] Rorty, *Objectivity, Relativism, and Truth: Philosophical Papers, Volume 1*, 16.

perspective, frames teaching practice into three categories of decisions (which, ostensibly, are applicable and prior to all teaching action):

(1) What content to teach,
(2) What the student will do to learn and to demonstrate learning has occurred, and
(3) What the teacher will do to facilitate the *acquisition* of that learning.[3]

Not surprisingly, there is an extensive literature critiquing this orientation (along with the series of associated realist, mechanical, objectifying metaphors that give it shape). Nevertheless, perhaps in part because our language seems to bias our thought and to mold our patterns of acting, most of current teaching practice appears intent to fit itself into this category.

The constructivist challenge, as might be expected, is founded on the conviction that one's dynamic and unique knowledge is the product of accumulated experience. Teaching, then, is not a matter of ensuring that pre-selected truths are acquired, but of facilitating the construction of knowledge through the creation of appropriate learning environments. In effect, teaching becomes a matter of *orchestrating* the learners' experiences rather than of transmitting knowledge, and the teacher is more centrally concerned with attending to emerging understandings than with providing unambiguous explications.

One might thus characterize "constructivist teaching" as a process of making sense of the sense students are making, founded as it is on the epistemological premise that learning has to do with the active and independent construction of meaning. In other words, the primary concern of the constructivist teacher is not "How might this best be told?" but "How can I tell if the learner has learned?"

This shift in orientation is a dramatic one, and it is not merely a movement from an outer authority (e.g., objective knowledge or an expert teacher) to an inner one (i.e., the learner's subjective conceptualizations). It is a move toward listening—albeit a listening that is cast in terms of a distanced attendance to the other's monologue rather than an intimate dialogic participation in the sense that is being made.

This conception of "constructivist teaching" is not without its problems, however. In particular, with constructivism's self-imposed focus on the individual's creation of subjective meanings (which, admittedly, occur in interactive settings), a host of other issues surrounding the social act of teaching tend to be pushed into the margins (or even disregarded). Topics such as human agency, our tremendous interactive capacities, and the moral dimension of teaching, for example, do not generally receive the same level of critical attention as the processes of sense-making—these are unproblematic givens. On one level, this "oversight" is quite appropriate: constructivism does not claim to be able to provide insight into such topics. On another level, however, this is a serious shortcoming. In failing to address these issues, proponents of constructivism are failing to interrogate the taken-for-granted that they bring to discussions of teaching.

A further problem with the notion of "constructivist teaching" arises from the fact that constructivism, as a theory of learning and knowing (i.e., an epistemology), can at best make us more aware of what we, as teachers, *cannot do*. Indeed, since a key tenet of constructivism is that there can be no such thing as an instructive act (i.e., in the

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[3] Madeline Hunter, *Mastery Teaching* (El Segundo, CA: TIP, 1986), 3. (emphasis added)

transmissive sense of the phrase), the teacher can only be considered in terms of a source of perturbations, the purposes of which are to systematically prod learners toward certain pre-specified understandings. Put differently, in conventional terms, *teaching* has to do with *causing* someone to learn something. (This definition is fairly standard and seems to work across most educational philosophies.) Yet *constructivism* begins with the premise that no one can *cause* anyone to learn anything in particular. "Constructivist teaching," then, is something of a paradox, if not a complete oxymoron.

It might be expected, then, as these notions have filtered their way from the theorist through the researcher to the teacher, they have collided with enough violence to create an unbridgeable theory-practice chasm. The teacher, who is charged with ensuring learners develop particular mechanical competencies is simultaneously stripped of any efficient means of doing so. Rather than enabling us to overcome some of the problems surrounding the teaching of mathematics, then, the constructivist "revolution" has served to exacerbate them.

A Critique

Just as the movement from objectivist to subjectivist accounts of learning and understanding amounted to little more than a transference of monologic authority, the shift from the conventional *teaching as telling* to the constructivist *teaching as orchestrating* amounts to little more than a renewed attempt to prescribe or control the learning that is to occur. That is, in spite of the insights offered by constructivist theorists, recent developments have only served to bolster the modernist desire to dictate outcomes—though the sanctioned means of achieving the desired ends are markedly different.

The problem, I would argue, arises from the constructivists' reluctance to step outside the "neutral" bounds of epistemology into the messier and more demanding realms of morality, ethics, identity, and being—ontology. As such, they have not just "gone along with," but have actually provided support for the continued definition of educational practice in terms of modernist ideals. The learner remains autonomous, coherent, and insulated; the curriculum continues to be prescribed, external, and controlled; the subject matter is still perceived as inert, unbiased, and valuable; understanding remains state-able, terminal, and personal. The preceding chapters represent my attempt to pull at, untangle, and re-weave each of these strands.

Continuing in that vein, I turn again to enactivist theory to seek an alternative understanding of teaching. My starting place is to invoke once again Varela's distinction between *prescription* (what is not allowed is forbidden) and *proscription* (what is not forbidden is allowed)—a shift which I believe might make it possible to overcome the inability of conventional learning theories to inform teaching. Briefly, enactivist teaching offers a way of distancing ourselves from the constrained consequences of modernist efforts to prescribe learning outcomes and to move toward a more proscriptive orientation whereby diverse possibilities are embraced as the teacher becomes an important interactive and co-emergent part of the learning context.

Current efforts to describe "constructivist teaching" have been crippled by the built-in requirement that they focus on the prescriptive "what is not allowed"—among those things forbidden (or deemed impossible) are pre-determined outcomes, transmissive acts (telling), shared understandings, and coupled action. If, however, we shift our focus to "what is not forbidden," and abandon attempts to control learning, then we are no longer compelled to base our teaching actions on as-yet unrealized endpoints—a practice

that forces us to privilege what we eventually want to achieve at the expense of what is currently happening.[4]

An Alternative

Enactivist teaching, then, focuses on the *now*—on emergent understandings, on immediate possibilities for action. Knowledge, rather than being understood in objective or subjective terms—whereby persons and their understandings are regarded as essentially isolated and autonomous—is recast as those patterns of acting that allow our structures to be coupled, thus entangling us in one another's existence and implicating us in one another's knowing. Teaching, in effect, comes to involve the presentation of occasions for *play* ("play" is understood as referring to the possibility for movement or 76ely aimless and childish activity that is described in psychology texts). The conventional relational, temporal, and spatial *bounds* of teaching are divested of their objective form and invested in terms of pedagogical sensitivities and ecological mindfulness. Efforts to locate mathematics in objects or subjects are replaced by an understanding that mathematics is neither inside nor outside, but *about* us.

In this conception, teaching can be neither about *telling* nor about *orchestrating*. Neither, however, are the acts of telling or orchestrating precluded in the enactivist frame. I, the teacher, can still tell; I, the teacher, can still orchestrate. However, it is the learner, and not I, who judges whether I have told or orchestrated. The teacher must thus be attentive to the consequences of her interventions, attuned to the moment-to-moment activity of the classroom, and inquiring into the possibilities of the spaces that present themselves. She must be listening.

In no way is this meant to suggest that the teacher must forego all hopes of promoting understandings on particular concepts (that is, those mandated by formal curriculum documents). Quite the contrary, by offering the possibility for joint action (rather than merely coordinated movement), a teaching founded on listening makes tenable the teacher's position between collective knowledge and individual understandings. Rather than attempting to serve as a conduit from the former to the latter, or to shape the latter into the former, the listening teacher, attentive to the play and interplay of both, moves back and forth between them.

Such intertextual movement is implicit in listening. Even in the most naive formulation, listening is necessarily dialogical, involving at the very least the intermingling of another's words with the text of my own experience. As such, on the

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[4] This critique of constructivist teaching must not be interpreted as suggesting that the constructivist framework is wrong or that it has little to offer. Quite the contrary, it is clear that any insight into the processes of learning will be valuable in discussions of teaching. (To this end, considerable preliminary work into the possibilities for constructivist teaching has been done by persons in mathematics education—with notable contributions coming from Jere Confrey, Robert. B. Davis, Les Steffe, and Paul Cobb. For a good introduction to some of this work, see Robert B. Davis, Carolyn A. Maher, and Nel Noddings, eds., *Constructivist Views on Teaching and Learning of Mathematics (Journal for Research in Mathematics Education, Monograph No. 4)* (Reston, VA: National Council of Teachers of Mathematics).)

The important point is that constructivism is not fundamentally concerned with teaching or with education. The particular domain of constructivism—that is, the individual's cognition—is an inadequate basis for redefining the socially-, historically-, and politically-saturated realm of educational practice. As such, we need to move beyond constructivism in order to consider the interpersonal, intentional space of teaching.

figurative level, listening offers us a powerful alternative to metaphors[5] of teaching which focus on the monological (such as "transmission," "telling," "voice," or "empowerment"). Similarly, as argued in Chapter 1, a listening orientation denies the possibility of rigid subject-object distinctions, reminding us that the issue of who we are is not separate from where we are, what we are doing, who we are with, and what we know.

More profoundly, because listening occurs (for the most part) in language, in listening we are called to a rich and many-leveled history of human participation in the world. Each sentence, each phrase, each word is meaningful not because a definition can be constructed, but because it is situated in the complex web of meaning-making action. It bears the trace of the past, a clue to the present, an anticipation of the future. The modern (and mathematical) tendency to "fix" words and to demand precise usage, in contrast, ignores the past, concretizes the present, and domesticates the future.

In this regard, the *listening* attitude (which questions the prejudices that shape our perceptions) might be contrasted with the act of *hearing* (which moves along unaware of the play of language). Going about our lives in a *hearing* mode compels us to exist in a modernist frame. A *listening* attitude offers the possibility of another way of being. Both *hearing* and *listening* attitudes are realized in language, but the first is held down by what *is* while the second is (to some limited extent; we can never step outside our language) freed to explore what *might be*. Hearing compels us to move in the fixed patterns of the world; listening invites us into the world's play. In the context of the conventional mathematics classroom, this distinction is an important one because there, it seems, most interaction is founded on hearing rather than on listening—that is, on an awareness of the other's presence, but not on an earnest desire to bring them forward; on definitions, but not on meanings.

Teaching Mathematics Backward

The point I wish to argue now, and the one which will serve to bridge this section to the next, is that the sequencing of instructional events in conventional textbook based mathematics classes—if we, as educators, are indeed concerned with promoting understandings, ensuring relevant program content, and being attentive to learners in other than a strictly intellectual capacity—is backward. Reducing it to its simplest terms, this sequence might be described as commencing with a formal concept and moving (via practice, application, proof, or some other "meaningful" activity) toward formulated understanding. It is thus that the typical class period begins with a brief lesson and the balance is given to seatwork.

Experience with this sort of structure is familiar to most in our culture. Some time ago, I was re-playing an audio-recording of a mathematics lesson for the purpose of locating and transcribing those teacher-student interactions that I thought might be informative. A colleague from English education entered my office and, after listening for only a moment, demanded that it be turned off. When I complied, he immediately assumed the role of the teacher and proceeded to "re-enact" the unheard remainder of the lesson. He did so with an uncanny accuracy, imitating not just the structure and the rhythm of the lesson, but also capturing the voice, the manner, and the bodily aspect of the teacher.

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[5] As I explain later, while there are some powerful consequences of interpreting teaching as listening metaphorically, for the most part my use of the phrase is quite literal.

The point here is that, based on his own experience with school mathematics (which was now long behind him), he was able to re-present the tone and structure of a "typical" lesson—a format that bears a certain resemblance to the popular image of the subject matter: predictable, hierarchical, precise, fragmented, self-contained; lacking rough edges, rhythm, fluidity, engagement, interaction. This is the mathematics of the textbook, and it is enacted not just in the lesson's structure, but the teacher's movements, the seating arrangements, and the resultant "understandings." Such fragmented curricula are the inevitable consequences of constructing courses out of textbooks, for, as Ong[6] has noted, writing and print isolate as they obviate immediate human interaction. It is in reference to their experiences in classrooms which are structured around textbooks that we hear teachers in staffrooms and students in hallways complaining that the other never listens.

A listening emphasis in teaching thus begins by shifting from the visual demands of the text to the auditory possibilities of dialogue and cooperative action. It disposes of the formal-concept-to-formulated-understanding sequence of conventional mathematics instruction and provides a space for mathematical play in which appropriate actions (whatever form they may take) are made the focal point. The issue here is not to provoke or to make possible learning actions; they are always and inevitably happening. Rather, the point is to open a space where such actions might be noticed.

Starting from a more enactive perspective, one might say that this reversed approach begins with enacted understandings—the *doing* rather than the *stating*—and then *moves*. Exactly where it moves depends on such complex factors as the structures of those present, the context, and what has been anticipated. It may move toward more formulated understandings, if such formulation is relevant to the play space or if it becomes part of a further exploration. It may simply move to other sorts of activities. This, of course, is not to say that we should just allow whatever might happen to happen, thus abandoning our responsibilities as teachers. Rather, it is to say that we cannot make others think the ways we think or know what we know, but we can create those openings where we can interactively and jointly move toward deeper understandings of a shared situation. The listening teacher's task in this sort of context is hardly that of a detached observer.

An illustrative example of this point was undertaken in a combined grades two and three class on the topics of multiplication. Copies of 10×10 grids were distributed and students were asked to color patterns of six squares in as many different ways as they could. Rectangles of dimensions 1×6, 2×3, 3×2, and 6×1 were quickly generated—and, in the conventional mathematics classroom, the production of these combinations would likely have prompted the teacher to choose another number to work with (since all the whole number factors of six had been identified). This teacher, however did not foreclose on the play, and soon some students had noticed, among other things, that twelve half-squares ($12 \times \frac{1}{2}$) and that twenty-four quarter-squares ($24 \times \frac{1}{4}$)—ideas that are normally reserved for students twice their age—also covered an area equivalent to six whole squares. Limited by their dexterity, they went no further, but a few were able to suggest other possibilities in the emerging series of combinations.

The teacher, in allowing the space for play, had opened up a mathematically rich space. She was, at this point, able to move in any of a number of directions. She might have introduced the formal concept of multiplication; she might have let students

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[6] Ong, *Orality and Literacy: The Technologizing of the Word.*

continue the same activity with other numbers; she might have changed topics completely (to fractions or geometry, both of which figure prominently in the program of studies). The key point here is that, rather than attempting to teach *toward* a narrow, specific, and pre-stated understanding, this sort of activity made it possible for her to teach *from* embodied understandings. Understanding was thus not a goal to achieve, but a quality to enact.

Listening—that attitude of openness to the possibilities that continuously present themselves—is essential to teaching in this conception.

Section B

Assessment

> The genuine teacher differs from the pupil only in that he can learn better and that he more genuinely wants to learn. In all teaching, the teacher learns the most.
> —Martin Heidegger[7]

The first few times I explained to my non-teaching friends that I was investigating the way that mathematics teachers listen, they reacted with laughter. For them, teaching in general, and mathematics teaching in particular, was understood to be about *not* listening.

What is really interesting is that this opinion is not limited to those outside of teaching. Some years ago, I took part in a mathematics curriculum and instruction course for practicing secondary teachers. Midway though the course, the professor asked the participants to "listen" to their students for one week and to bring to the next class some brief transcriptions of what they had heard. They returned the following week with comments like, "I listened but couldn't pick anything out," and, "My lessons just weren't conducive to doing that sort of thing this week." It appeared not only that these teachers' listening abilities were inactive while instructing, but that they perceived no reason to listen in the first place.

I was surprised at this event, particularly because a major focus of the course had been an exploration of the implications of constructivism for teaching. We had established, I thought, that teachers could not *cause* someone else to learn anything in particular; nor could they know if students had indeed learned. (The learner alone can make such determinations.) The only person that the "constructivist teacher" can be sure has been taught, then, is herself. Here the teacher must be the learner, and hence her responsibility to attend—to listen to learner's as they re-present their understandings—far outstrips her role in telling and managing. Yet, despite a week's effort, the teachers in the course seemed not to heard (learned) anything of substance from their students.

In this section I look at the issue of "assessment" (i.e., questioning, testing, and examining) as a route to understanding why these teachers were unable to listen. It is my contention that the technocratic rationality which underlies current comparison testing, standardized examinations, and other grading practices, is one of the major roadblocks to a more interactive and attentive mode of teaching.

Implementation versus Education

A preliminary point to be re-emphasized is that *listening* is not a technology to be implemented, it is an orientation—a way of standing in the world—that is enacted in a given setting. It thus cannot be legislated; it must simply be part of who you are as a teacher. This is not to say that the capacity to listen cannot be developed; but it is meant to suggest that one cannot become a listener simply by deciding to or planning to listen. The movement from listening to the evening news to standing in the world as a listener is not a matter of a personal resolve.

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[7] Heidegger, *Basic Writings*, 252.

My thesis has been that the preparation necessary for the listening teacher is broad and uncomfortably deep. It must cut across and through conceptions of knowledge, education, cognition, and teaching. Omitting any one of these elements, I contend, would seriously constrain one's listening. If we were to hold, for example, that the learning of mathematics was a matter of mastering pre-selected and universal truths, then the purpose of listening to learners would be strictly to diagnose and to remediate difficulties. Sadly, altogether too often, it seems that if the teacher attends at all in the mathematics classroom, it is in this "somewhat damping manner of listening only to correct."[8]

But if we hold that the learning of mathematics involves something other that the acquisition or mastery of knowledge, then listening takes on a quite different relevance. We become interested in students' interpretations of ideas and we grow more aware of how these interpretations are tangled in the webs of their existences. Thus our listening becomes a truly hermeneutic activity, one that requires the virtues of openness, humility, caution, and trust. (Indeed, as the oft-heard phrases "I should have listened" and "She's worth listening to" suggest, sincere listening is often equated with trust and belief in another.) In other words, our listening is a part of who we are, and the capacity to listen is not so much related to a desire to do so as it is to an inability to not do so. Just as we are unable to turn off our hearing, the listener "can't help but listen."

Questioning Questioning

The listener is oriented toward gaining a fuller understanding and is thus vigilant to the fallibility of interpretation. This is why the listener cannot be held silent; listening is itself a kind of speaking; it is a manner of probing and checking emerging understandings. As Gadamer explains, one *questions*, "one does not try to argue the other person down but . . . one really considers the weight of the other's opinion."[9] Within the conversation, this manner of speakingly-listening is expected. Indeed, we can only know if the other is listening if she responds in some way—and so we worry she is daydreaming or misunderstanding when she falls silent. Listening is not a solitary act, it is a reciprocal engagement.

Gadamer contrasts the manner of hermeneutic questioning that underlies listening with two other modes: classroom questioning[10] and rhetorical questioning, both of which he suggests lack the openness required of a "true question." The classroom question is one that lacks a questioner (i.e., the person asking already knows the answer, and so the purpose of asking the classroom question is not to interrogate or investigate an issue; rather, its purpose is toward maintaining students' attention or to converting their performances into summative grades). The rhetorical question lacks both questioner and answerer.

Whether a question is hermeneutic, classroom, or rhetorical is dependent not upon the manner in which it is phrased, but on the attitude of the questioner. Consider, for example, the question, "Where might we use positive and negative numbers?" Posed in a conventional mathematics classroom, the teacher would likely allow for a very limited

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[8] John Wyndham, *The Chrysalids* (London, Penguin Books, 1955), 69.

[9] Gadamer, *Truth and Method*, 367.

[10] Gadamer uses the phrase "pedagogical questioning" instead of "classroom questioning." Because "pedagogical" is being used in quite another manner in this text, I have elected to make this nominal change.

range of acceptable responses, and this point was well-illustrated in an introductory lesson on the topic:

> Teacher: Where might we use positive and negative numbers? [*He draws a T-chart on the chalkboard, placing a "+" in the upper left and a "–" in the upper right.*] First, we call this "positive" [*pointing at the "+" and writing "positive" beneath it.*] If you move this way [*walking and pointing forward*], this is positive. What am I doing?
>
> Students: [*calling out*] Forward; Walking forward; Moving.
>
> Teacher: Walking forward [*adding "forward" under "positive"*]. Okay, if I climb a hill, what am I doing?
>
> Students: Up; Climbing; Upwards; Climbing up; Ascending; Rising.
>
> Teacher: Upwards [*adding "upward" to the list*]. Can you think of any other words that mean "positive"? Like temperatures rising [*adding "rising"*]. Any others?

Here the questioning is really a thinly disguised *telling*, where the participants are playing a game of "guess-what-I'm-thinking." The teacher is not listening to the responses; he is clearly selecting those words that he wants to hear from among a chorus of answers. In this case, all answers are assessed as either right or wrong—which is to say, a response is either what the teacher wants to hear (correct, and therefore acknowledged) or what the teacher feels is irrelevant (mistaken, and hence ignored.) These are the sorts of questions critiqued by Douglas Barnes:

> Much teaching leaves the pupils dependent not on publicly established systems of knowledge (if such exist) but on quite trivial preconceptions set up arbitrarily either on the spur of the moment, or when the teacher planned the lesson during the previous evening. *This reduces the part played by the pupils to a kind of guesswork in which they try to home in upon the teacher's signals about what kind of answer is acceptable.*[11]

The rapid-fire elicitation of rote-responses is the dominant form of questioning in many mathematics classrooms. These questions are asked with little interest and framed by "the teacher's own, implicit association of thought and reference."[12] They thus effectively close down the possibility for much mathematical thought. (Small wonder that critical theorists regard teacher-student relationships as more political than pedagogical.)

The same question, "Where might we use positive and negative numbers?" might easily have become a hermeneutic question—an open one—the essential quality of which is that the answer not be settled. The questioner participates in the questionability of what is questioned; there is some indeterminacy. It is thus that a hermeneutic question is not about reporting on truth, but about creating it. It is this sort of question that might "break open" a lesson—but only if the person who has posed it is genuinely participating in the question by listening to the actions it provokes.

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[11] Douglas R. Barnes, *From Communication to Curriculum* (Hammondworth, GB: Penguin, 1976), 179. (emphasis added)

[12] Edwards and Mercer, *Common Knowledge: The Development of Understanding in the Classroom*, 30.

Such questioning is an art. It moves beyond the classroom question that calls only for a re-presentation of something that has already been established. The hermeneutic or open question offers the possibility of coming upon something new; it presents possibilities. As such, the emergence of the question is not really under the control of the questioner. Rather, it conducts the person who, once attuned to what is being asked, cannot help but puzzle over it or wonder about it.

In effect, then, the essential difference between the classroom question and the hermeneutic question is that the former has become a substitute for listening while the latter exists only in listening. That students' responses to classroom questions are generally not listened to is evidenced by the fact that they generally have little or no effect on the course of a lesson. The same explanations are still given (and re-given); the same exercises are still assigned. They are questions whose relevance is determined in advance by their connection to a pre-stated objective, by their place in some technologized taxonomy, or by the blind (and deaf) faith that they will motivate learners. For the premise underlying this mode of inquiry (or, rather, *inquisition*) is that knowledge, understanding, and identity are states that can be accessed and assessed through the terminal question.

The hermeneutic or open question, however, is founded on the belief that "the path of all knowledge leads through the question."[13] The question does not *follow* learning, it precedes it. It points to the not yet known and to the wondrous. It is thus that the hermeneutic question, like the listening and the understanding that flow alongside it, is more an attitude than an object. It is the sort of question that is quickly transformed by those asked, taking the form of new questions. Conventional worries about how to make the subject relevant and how to motivate learners fade to irrelevance in the presence of a question intended to engage rather than to assess.

Testing Testing

Gadamer also makes use of the term "testing" in his description of the sort of probing that is necessary when the listener is seeking a deeper understanding. It is a testing, then, not so much of what the speaker knows, but of one's own hearing. However, like questioning, testing has assumed a very different character in the modern mathematics classroom.

The term *test* has a long and rich history, and it shares its roots with *text* and *texture*. The current (schooling) sense of a (usually written) examination is a modern artifact that belies its origins. For in the regime of testing that surrounds mathematics learning, there is very little concern for the texture of one's understandings, let alone for the text from which they arise.

Testing, like questioning, has become a substitute for listening. Worse yet, the pervasive presence of standardized mathematics examinations at levels from the classroom to the United Nations has created a performance requirement that stands between teacher and learner—one that, from the vantage point of the teacher administering the tests, not only obviates the need to listen, but militates against it. The instrumental proficiency necessary for success on such tests, at first glance, seems incommensurate with an orientation to teaching that is in deliberate opposition to the mechanical rationality underlying the whole business.

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[13] Gadamer, *Truth and Method*, 363.

It is not my primary purpose here to critique the conventional emphasis on testing. Rather, my intentions are to comment on its uni-dimensional character and to explore alternatives. Conventional testing can do very little, situated as it is at the end of an assembly line. At best, it provides some information about who could do what on a certain day in a certain place. For the teacher, it might help to pinpoint those skills that require further refinement; for the learner, it might reveal gaps in formulated understandings. But in neither case does it promote communicative action, for that is simply not its purpose. The test is usually administered *after* (instead of during) the study of a given topic; the test is usually stripped of a context of action; the test is usually designed to reveal little about understanding in the first place.

Michel Foucault provides a scathing critique of the test as a mechanism of education. In his words, it "compares, differentiates, hierarchizes, homogenizes, excludes. In short it *normalizes*."[14] In his analysis of the formal test or examination, Foucault describes how this device subjects its subjects (where both verb and noun forms of "subjects" are used polysemously). As Gallagher elaborates,

> [Through such procedures as the examination, education] acts as a machine into which we put nonsubjects (children who live on the principle of play) and by which we produce *subjects*, in every sense of that word. These procedures . . . (a) . . . objectify their subjects by making them *visible* in the light of certain measuring criteria; (b) they document their subjects bestowing upon them a personal history which captures and fixes them; and (c) they define each individual as a "case."[15]

In short, the formal test, as an ostensibly educative tool, serves to distance one person from another, to break play from knowledge, and to assign in non-negotiable terms an identity to the learner.

And so, if we are to seek an alternative to teaching, we must also seek an alternative to the testing regime that defines it—challenging first such assumptions as the comparability of student performance. As Elliot Eisner explains,

> Educational evaluation and measurement have been predicated on the need to compare students with each other or with a known criterion. . . . [Such] a premise is not a necessary condition for any kind of evaluation. As our premises change so that we are open to forms that are distinctive, we will be in a much better position to develop evaluation practices that recognize the cultivation of productive idiosyncrasy as an important educational outcome and thus to honor it in assessment.[16]

It is my contention that a listening orientation to teaching offers a means of moving toward what Eisner is calling for. It begins by returning "assessment" to its etymological roots of "sitting beside."

A listening emphasis in teaching points to a need to explore alternative frameworks for testing. A starting place might be a redefinition of the word "testing" to include a *scrutinizing of the teacher's interpretations* of students' work—a testing of one's hearing, as it were. The purpose of testing here would be to facilitate classroom

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[14] Michel Foucault, *Discipline and Punish: The Birth of Prison*, trans. Alan Sheridan (New York: Pantheon Books, 1977), 183.

[15] Gallagher, *Hermeneutics and Education*, 298.

[16] Eisner, "Forms of Understanding and the Future of Educational Research," 7.

interaction, not strictly to assess summatively and retrospectively its effectiveness (nor, as Foucault has argued, to exert one's normalizing authority). One tests to check on *one's own* emerging conceptualizations. Testing, then, is aimed as much at the teacher's prejudices as at the learner's understandings. It occurs in the activity of learning, not after it.

It is important to note that this orientation to testing, arising as it does from an emphasis on listening, is not necessarily incompatible with the current desire to raise "achievement standards." Carpenter and Fennema, for example, reporting on the relative effectiveness of particular mathematics teachers, conclude that "listening to their students was the critical factor"[17] in promoting increased competencies. Such evidence serves to support the contention that an attentiveness to interpersonal relationships—and not to better management, clearer explanations, increased accountability, or more elaborate technologies—leads to better understandings.

The Contingencies of the Classroom

Both questioning and testing, then, are fundamental to the orienting and the enabling of learning. They are critical to the classroom because of the uncertainties of the teaching process. In effect, a teaching guided by listening (which relies on questioning and testing) is a teaching which embraces the contingencies of existence—the likely but not certain, the dependent but not predictable. It is, in Gallagher's words, a recognition "that we cannot avoid ambiguity and therefore must not deny its operation but find a way to live with it without inflating its effect."[18]

Teaching, in this way, might be thought of as an attempt to "condition ambiguity"—to expose the contingencies of current understandings. Such "conditioning" can be violent and unsettling, or it can be undertaken with sensitivity and tact. It can be imposed, or it can be drawn from immediate experience. It can try to control, or it can embrace the complexity and variation that is present in any social context.

Put differently, in teaching we can either ignore or embrace the watershed moments—those unpredictable and unplannable events that are always happening in our classrooms—that conduct our learning. We can continue in our attempts to structure potentially rich settings and to point fervently to that richness, or we can offer such settings and pursue those elements that capture the attention of learners. We can focus our actions on conducting our lessons, or we can allow our listening to conduct our actions.

Following Weinsheimer,[19] I will be using the word "hap" to refer to these watershed moments. "Hap" is an archaic word meaning "event"—and referring particularly to those events that come to be associated with good fortune. It is the root of may familiar words, including *happen, happy, perhaps, mishap*, and *happenstance*. Using the notion to critique conventional mathematics education, one might say the efforts to formulate a program of studies and to prescribe appropriate instructional methods are tightly linked to the desire for a *hap*-less curriculum wherein the teacher is able to prescribe all learning and to foresee every possible contingency. Such desire has led to an

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[17] Thomas Carpenter and Elizabeth Fennema, "Cognitively guided instruction: Building on the knowledge of students and teachers," in *Researching Educational Reform: The Case of School Mathematics in the United States*, ed. W. Secada (a special issue of *International Journal of Educational Research*, 1992): 457-470, 467-8.

[18] Gallagher, *Hermeneutics and Education*, 343.

[19] Weinsheimer, *Gadamer's Hermeneutics: A Reading of "Truth and Method."*

impersonalized (or, perhaps a more appropriate term would be "depersonalizing") model of mathematics education. Pre-stated objectives have eclipsed the issue of personal interest, and "fool-proof" explanations have eliminated the need for personal insight. In such mechanized curricula, the hap is something to be ignored or, better yet, avoided entirely.

It is argued here, however, that the hap is the center-point of enactivist teaching. Such teaching is an attendance to the unexpected consequence, to the sudden insight, to the inexplicable interest that is conditioned or occasioned by the teacher's actions. The hap may be anticipated (and, because of this possibility, the teacher has a responsibility to consider what might happen in a given setting), but will more likely be a matter of happenstance.

The final section of this chapter I devote to a few descriptions of these ideas, framed by a closer examination of the classroom account that was used to end the preceding chapter.

Section C

Mathematics Teaching as Listening

> From the teacher's end it boils down to whether or not she is a good conversationalist; whether or not she has the gift or the wisdom to listen to another; the ability to draw out and preserve that other's line of thought.
> —Sylvia Ashton-Warner[20]

I return in this section to the classroom example that was presented at the end of the preceding chapter. There I used it to point toward the possibilities of a play-full classroom. Here I use it to recap the main points of this text and to re-assert my underlying premise: that mathematics teaching might better be understood in terms of listening than according to the visual, mechanical, and economic frames that currently dominate discussions in the field. In what follows, the anecdote has been broken into three pieces, each of which serves to head a section dealing with one of the three horizontal strands of this text (i.e., the subject matter, formal education, and cognition).

My intention here is neither to idolize a particular teacher nor to romanticize a specific teaching moment. Rather, it is my hope that the manner in which the particular is embedded in the general, the way the general is enacted in the particular, the way we move back-and-forth between the appropriated (the given) and the transformed (the interpreted), all become audible through the structure provided. The tone and the issues are thus deliberately understated, for the goal is not to re-present or to re-synthesize but to re-mind. This is not a conclusion; it is not the closing statement of an argument. It is, rather, my attempt to re-think teaching, and in it I am guided by the advice of Richard Rorty: "a talent for speaking differently rather than for arguing well is the chief instrument of cultural change."[21]

I aim neither to convince nor to identify the "educational implications" of my study. My purpose is, rather, to try to speak differently about teaching. The results are not spectacular but, consistent with my sonorous theme, I believe them to be sound.

Listening *in*—Subject Matter

> Tom wanders among his grade seven students as they play with their *Fraction Kits*, sheets of paper cut into halves, thirds, and other fractional pieces. He has posed a question to the class: "What can you say about three fourths?"
>
> The class members are noisy and active. Groups are huddled together, most of them using the kits to devise methods of covering three fourths of a piece of paper. As Tom moves about the room, he listens, he questions, he requests that students display their work on the chalkboard.

To the eye, there is nothing new here. Progressive, perhaps, but not new. To the ear, however, this is not a typical classroom.

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[20] Sylvia Aston-Warner, *Teacher* (New York: Simon and Schuster, Inc., 1963), 58.

[21] Rorty, *Contingency, Irony, and Solidarity*, 7.

Consider for example, the sound of the mathematics. It does not have the tone of formal mechanized knowledge—there is no droning repetitive beat guiding the actions here. Instead of the sterile and neatly dissected facts of the textbook, this mathematics is messy and tangled. It is an active mathematics that orders, arranges, reasons, and suggests. It is a participatory mathematics that both allows for and demands engaged action with one another and with the objects of one's world—in both cases, a reciprocal and a mutually specifying process is occurring. Words are knit together into a wider integrated knowing; actions prompt other actions as they alter the presented world.

The mathematics is a ground for exploration. There is some creation, some discovery—but the line between constructed and discovered is not clear in the conversation. That is to say, there is a rhythm to the movement in this classroom that passes beyond the merely coordinated actions of other modern school settings. The subject matter here is neither outside the learners nor in any one of them, but *about* them. It flows about them, it tells about them.

Here we find the teacher positioned between the collective mathematics knowledge of the culture and the emergent mathematical knowings of the individual. The teacher is *listening in*—attuned—to both and, in this listening, is implicated as a full participant in both. In this way, he is enacting an ecological mathematics. It is a mathematics that is not held as distinct from nor thought to be superior to other disciplines and other modes of reasoning. These learners are not expected to leave behind (or suppress) experiences from other contexts—rather, they are offered a space to re-present them and, in so-doing, to re-interpret what was done and what is being done. And these learners are not held apart by a rigid seating pattern or by the irrelevance of an uninteresting or closed activity.

And so this teacher's *listening in* is not an attending that occurs at a distance. It is not an eavesdropping or a surveillance but an action that locates him in a complex web of existence—caught in intertwining and evolving lines of text from which one cannot extricate oneself. The teacher is not guiding a sight-seeing tour through a thoroughly mapped-out region, but is dwelling in, with, and through the complexity and ambiguity of emergent knowings. A full participant in the learning that is occurring, the teacher is part of the simultaneous transformation of knower and known, culture and mathematics.

What is the "curriculum" here? It is clearly not the narrow, instrumental, prescriptive program of studies that one finds represented in texts and enacted in most classrooms. And implementing the curriculum is not a matter of ensuring that learners achieve some set of predetermined technical competencies.

Rather, the static-ness of the curriculum has been dissolved into the fluidity of *currere*—from following a pre-structured path to laying down a path in walking. The steps taken are thus more tentative and more explorative, for the attention is on the negotiation of the terrain rather than on the efficient passage through it. And so, there is no terminating point; no one says, "I'm done. What do I do now?"

But the mandated curriculum has not been cast aside in this dissolution. Reference to the relevant objectives or topics of study can easily be made (although it would be impossible to isolate a single concept as the textbook does). The important point here is that *listening* does not imply an abandonment of the official program of study in a misguided attempt to pursue the richness of student action. Far from abdicating responsibility, in allowing for a more fluid curriculum, the listening teacher takes on a critical response-ability. It is not a listening that follows, but a listening that leads. In its attentiveness, it prompts and encourages—it shapes by questioning or by gesture. Put

differently, the listener selects in the speaker what is said and what is not said through a differential attendance. It is thus that the criticism, "You're not listening," is more than an accusation of non-attendance. It is an accusation of non-participation.

But how does one plan for such teaching? An answer, I think, is revealed in the question that has oriented this class's activity: "What can you say about three fourths?" This is a proscriptive question, not a prescriptive one.

It is a question, then, that presents possibilities for action. Contrasting it with the more traditional "lesson concept," Tom's question is not one that focuses on an isolated or fragmented idea. Rather, it is an opening for learners to devise questions of their own, to add, subtract, multiply, compare—all at levels of complexity that, in this case, far surpass the recommendations of the provincial program of studies.

The setting is thus one that is not so much "planned" as it is "anticipated." What sorts of mathematical investigating *might* (versus *will*) students undertake, given their backgrounds and the materials at hand? What sorts of prior experiences are necessary for learners to be productive (versus reproductive) in this context? These are the types of questions involved in this anticipating. As such, it is a "planning" founded on a broad familiarity with the mathematics as well as an extensive knowledge of the learners. "What can you say about three fourths?" is not a question that can be given to any group of students, nor is it a question that can be simply adapted to any topic of study.

Such anticipating is thus only possible through a history of *listening in*—further underscoring that *listening in* does not distance the teacher, nor does it exclude him from the action. Rather, even while standing apart from a group of busy learners, Tom is an integral element in their activity through a *listening in* that enters in, that participates.

Listening *for*—Education

> Not everyone is finished when he calls them to a discussion of what they've done. Interesting insights and possible avenues for further investigation are presented. Sarah asks a question: "How many different ways can you make three fourths using the pieces from the kits?"
>
> "I don't know. I'm not sure if anyone knows," is Tom's response. "I wonder if we can figure that out."
>
> Activity resumes as the groups produce five, six, nine, eleven, answers. As Tom listens in on one group, he notices that Greg is using the charts differently than his classmates: as a generative tool rather than merely as a recording device. With it he can determine ALL the possibilities for three fourths—and for any other fraction quantity—quickly and efficiently.

Auditory perception is not a simple matter of recording the sounds that reach our ears and replaying them in our minds. In fact, for the most part, perception flows in the other direction. What we hear is primarily a matter of what we expect or anticipate—what we are *listening for*. As an awareness of this phenomenon and of its implications are developed, we open the possibility for a deeper listening—one that still *listens for*, since we can never step outside of our prejudices, but one which listens for possibilities and not for just actualities. It is a listening that questions and that entertains questions. It is a listening founded on the fluidity of our knowledge rather than on its rigidity. It is a listening that, in the words of Herbert Marcuse, is "not only the basis for the

epistemological constitution of reality, but also for its transformation, its subversion in the interest of liberation."[22]

Consider Tom's answer to Sarah's question. "I don't know. I'm not sure if anyone knows. I wonder if we can figure that out." It is a statement that locates him, his students, and their mathematics in the tentative, inquisitive space of transformation. It is a space away from the surety of infallible knowledge, but one which does not surrender to the insecurity of the unknowable. It is thus a space for thought and action, of appropriation and transformation, of *listening for* that which speaks to us here and now out of our traditions. It is the space of education.

"I don't know," is not merely an expression of one teacher's personal limits. It is a stance in teaching. The essential quality here is not to know everything, but to be *listening for* those things that are not known. The teacher's task is not to transmit, not to manipulate learners into performing in desired ways by pretending to not have an answer. Rather, it is to seek out those spaces where *all* is not yet known. It is there that the mathematics can serve to open up the world. It can begin to educe and to present, moving beyond its current classroom functions of reducing and representing.

"I'm not sure if anyone knows," moves the discussion into the realm of the cultural. Sarah's question may have appeared a simple one, but it has suddenly uncovered an issue that is not settled. In this movement from the particular mathematics classroom to the general mathematics community, learners are situated between the actual and the possible and between past and future.

"I wonder if we can figure that out," is an invitation to hypothesize, to negotiate. It is a call not so much to do mathematics as it is to think mathematically. It is a statement on the role of the student. Learning is about neither acquisition nor subjective construction. It is about joint exploration, interactive investigation—play-full conversation.

All of this has become possible because of the secure relational space of this classroom. The teacher's interest in these children extends far beyond a concern for their academic competence. His is a pedagogical concern for their well-being, and it is revealed in his every movement. He embodies the qualities of hope, trust, patience, and humor—and nowhere is this attitude more apparent than when he leans in to listen. It is a listening that moves outward to engulf the speaker, to bring the two of them together in an interactive unity.

The educative potential of his listening is revealed in Greg's actions. In another classroom, Greg might have finished the assigned exercises and moved on to free-reading or some other activity. Here he is pushing out the bounds of his own understandings, looking for patterns inside patterns.

Listening *to*—Cognition

> Noting that Jake, Greg's partner, is copying Greg's responses without really participating, Tom asks Greg to explain his reasoning. Greg complies, but it is clear that Jake doesn't understand. However, the event does prompt him to stop copying and to return to manipulating the pieces of the kit.

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[22] Herbert Marcuse, *Counter-Revolution and Revolt* (Boston, MA: Beacon Press, 1972), 71.

Moments later, Tom feels a small poke in this back. He turns to find Jake holding up his chart. On it an interesting pattern has been recorded.

"I know everything about '1'," Jake announces.

Jake and Greg had found themselves in the same group because of the Cooperative Learning dictum that the "weaker" students will benefit from being paired with the "stronger" ones. In this situation, Greg had come across something and was eager to play with it; Jake was in his usual spot: not quite "getting" it, trying to keep up by copying someone else's work. With a history of difficulty in classroom mathematics, Jake was unable to cope with Greg's approach and (as later analysis would indicate) was having other troubles with the assigned task as well.

What is abundantly clear in this instance is that knowing has nothing to do with the computer-influenced notion of inputting, processing, and outputting data, for Jake would have had ample opportunity during class to "acquire" the small amount of "information" needed to go about determining different combinations for three fourths. The impact of the teacher's intervention here was not to alert Jake to his lack of knowledge, but to the inappropriateness of his actions.

That is, the Fraction Kits setting was designed as an explorative and interactive space—in effect, a space where one's knowing was inextricable from one's doing. Most importantly, it was a space that invited a range of mathematical actions—from the less sophisticated (in formal mathematics terms) re-arranging of paper pieces to more formal and abstract activities such as Greg's chart work. And so, in this setting, the measure of knowing was not the number of correct answers generated, but the appropriateness of one's actions. As such, Jake was demonstrating an inappropriate understanding at first.

But that quickly changed. Jake returned to the kit and renegotiated the difficulty of the question by changing the assigned total of "three fourths" to the more straightforward total of "one." Very quickly thereafter he realized that the charts could be generative devices—a realization that, among his classmates, was shared only by Greg. Jake also knew that his new understanding was something important—something as-yet unseen by the others. His voice was brimming with pride as he announced that he knew "everything about '1'."

And, in this instance, it was clear that "who one is" is not independent of "what one knows." At the start of this unit of study, Jake was the weakest student in class. But this event, along with many others, contributed to a school-wide honor of "Most Improved Student" by the end of the reporting period. Jake became, in this classroom at least, a fundamentally different person—one who could do math, could know math, could be a mathematizer. Jake, in this instance, demonstrated that the measure of learning is not an increased store of knowledge, but a broadened horizon. Learning was an event of being.

He also demonstrated that "facts" or "truths" are immediate, practical, and transitory. By a more rigorous standard, he could never know "everything about one." Nor could anyone. But his announcement, like the understanding on which it was founded, was truthful, and its facticity was revealed in the possibilities it presented. Further, the *truth* was neither in Jake nor outside of him. It was a dialogical truth of his action in that setting. And it was a truth that came about because someone was *listening to* him.

Understanding and meaning, in this setting, are qualities that are inherent in student actions and which can not be separated from their actions. They are thus

diverse—and perhaps the most critical feature of understandings is the possibility for such diversity. Just as genetic diversity is an essential quality for the viability of a species, so conceptual diversity is a critical element in any learning situation. Much in contrast to a conventional classroom, where the desired goal might be described in terms of achieving uniform competencies and standard understandings, here the focus is on an openness to—a *listening to*—the possible. Tom's request to Greg that he explain his charts, for example, was not an attempt to get Jake to imitate or to think like his partner. Rather, it was more an expression of hope that Jake would, as he soon did, realize that behaviors other than copying were possible. What Jake did do, to this end, was unpredictable and unrepeatable—an instance of diversity—and therefore generative of other possibilities.

This diversity places significant demands on the listening teacher who must constantly negotiate the tension between what was anticipated and what is happening. What it points to strongly is the need for a common repertoire of experience among learners on which to base actions and interpretations. Lacking such a foundation for shared meanings, one's interaction with another is severely constrained. This is the place of the Fraction Kits activity in this setting—a space for joint action, common language, and shared understandings; a space which not only demands, but facilitates, listening.

More importantly, perhaps, it is a space wherein the unformulated understandings that are enacted in every movement are not contemned. Rather, they are considered alongside those understandings that have been formally re-presented for inspection, confirmation, and revision. Formulated and unformulated knowings are thus understood not independently but in terms of their reciprocal (mutual) affect. The teacher listens to not just the words that are spoken, but to the actions that precede, accompany, and follow those actions.

Put differently, the Fraction Kit activity presents a space of play—of acting, of imagining, of moving. As Jake's investigation of "one" illustrates, it is a space that allows room to negotiate the given task, to turn it over, to play with it. (Such play is almost impossible when questions for which the answers are predetermined form the basis of student action.) In a setting such as that occasioned by the Fraction Kits, there must also be play in the teacher's listening. Lacking such play, for example, it is easy to imagine the response of a non-listening teacher to Jake's new understanding of "one": "Nice, but you're doing the wrong thing."

Play, as Gadamer has pointed out, opens the space of transformation. In that space, subjectivity is put aside and, as the self is re-membered afterward, it is changed. It might be said, then, that play engages, embraces, and encompasses. In playing—in moving, in understanding—our beings come to form. The teacher, then, cannot be considered apart from the learner's play. She is not a director or a facilitator, but an important part of the interactive setting—caught up in the play—*listening to* whatever might be happening.

Listening—Teaching

As teachers, we are continually confronted with claims that this textbook is better, this method is more effective, these activities are the best. There is a perpetual search for the optimum—for the highest score, the fastest procedure, the surest approach.

As if it were possible to attain such goals.

The sort of teaching that emerges from a listening orientation might be described as "good enough." It makes no attempt at optimization. Instead, the listening teacher works with the contingencies of the particular classroom setting. It is founded on the realizations that no learning outcome can be prescribed, no active setting can be controlled. But neither must we forego attempts to influence (or fail to acknowledge our influence on) what might come about. The key to teaching, in this conception, is to present a space for action and then to be present to participate in—and through this participation, to shape—whatever happens.

In the vignette above, Tom could have in no way foreseen the mathematics that would arise from Greg's or Jake's actions. What these learners did was not "caused" by what Tom did; their actions were, rather, determined by their own structures. Tom merely provided an occasion for them. While he may have anticipated what might occur (based on his familiarity with the learners and the situation), he could in no way have predicted or controlled (that is, caused) the outcome. It is precisely because the actions that the teacher occasions cannot be anticipated that one is compelled to listen: attuned to, in sync with, and following the rhythm of their actions. The teacher, in this conception, is not responsible for motivating learners. Rather, he takes advantage of their own (structurally determined) playful motivations.

The learning occasion described was a powerful one, especially when compared to more algorithmically-based settings, but it was one which cannot be considered in terms of optimization. Rather, it was one that worked. Similarly, the measure of the resulting understandings can not be discussed in terms of perfection, but must be considered in terms of their localness and particularity to the occasion. They, like the understandings that help us to maintain our viability in any other context, were *good enough*.

That is to say, they *fitted*. This criterion of fitness, borrowed from a post-Darwinian conception of evolutionary theory by which the logic of survival is proscriptive (requiring an adequate fit) rather than prescriptive (requiring an optimal fit), calls for a teaching orientation that is not just attuned to, but able to be shaped by, the learning setting. It must be able to maintain its fitness—that is, it must be listening to the events that are about it.

In listening, I am able to bring the insights that emerge from the mass of my experiences onto the developing conceptualizations of learners—not for the purposes of imposing "truth" or assessing performance, but to assist them in exploring and affecting (conversing with) the world. I need not downplay my wider experience, because it is precisely that experience that enables my listening. But, as I teach, I must be prepared to interrogate and to reformulate what I know, for, in order to listen, I am compelled to open my understandings and my self to transformation. If I do not, then I am keeping myself apart from—closing myself to—the very learning attitude that I am demanding of students. The listening teacher is thus the person who is able to forget the unified, coherent, self and to enter the conversation.

The role of the listening teacher is neither *telling* nor *orchestrating*—although proponents of both traditional and constructivist perspectives would likely see these elements to be woven into the teacher's actions. In Tom's case, for example, the initial structuring of the activity and the selection of the orienting question was very directive. At the same time, the active and explorative setting might be described by some as "constructivist."

But his departure from these orientations is clear, first in his reluctance to rigidly pre-determine what would be considered "appropriate" actions or understandings, and second in his refusal to privilege either the "voice" of the learner or that of established knowledge. Rather, a space was opened for their dialogue, and Tom was an unabashed participant in the conversation that emerged. His role was thus to provide occasions that would provoke and support on-going actions, and to play it by ear from there. In effect, he was creating the conditions for ambiguity—pointing at the gaps in knowing—while providing a space to negotiate the play in one's understandings. It is thus that Tom opened the possibility for and acted upon several *haps* (including, in the brief account provided, Sarah's question, Greg's charts, Jake's insight) which, together, form a complex web of events that could never have been deliberately provoked. But, with the prodding of an attentive, attuned, and knowledgeable—that is, listening—teacher, an ever-expanding sphere of possibilities was presented.

Of course, this does not preclude the possibility of effective teaching occurring in conventional mathematics lessons. Rather, it shifts the location of teaching to the opportunity for thoughtful action that is occasioned by the lesson and away from the formal lesson presentation. Teaching, then, does not occur in the well-articulated 10-minute explanation nor in the carefully-selected and thoughtfully-structured set of exercises that follow. Rather, teaching takes place in mutually specifying dynamics of the teacher-student relationship. There the teacher participates in the play-full learning of the student; there the teacher, so-disposed, can listen.

In sum, then, an enacted curriculum is one that should be planned, but not pre-determined. It occurs as a teacher occasions and interprets student actions. At times the teacher's actions are deliberate (that is, based on careful analysis and thoughtful decisions), but, for the most part, her actions are simply a consequence of the way she stands in the world. They are actions that arise from

a genuine curiosity in the mathematical subject matter of the setting (Chapter 2),

awarenesses of the place of education and where teaching takes place (Chapter 3),

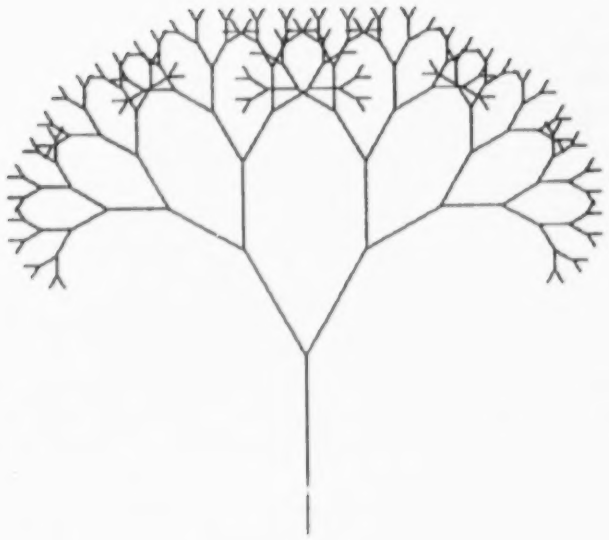
a hermeneutic attitude toward the playful learning actions of students (Chapter 4).

In the example given, Tom embodied mathematics, lived his pedagogy, and enacted his interest in learner understandings. His teaching, then, was not merely informed by his listening; it was in itself an act of listening. In listening he positioned himself amid the dynamic interplay of evolving meanings and established understandings.

In listening, he created a place for mathematics teaching to happen.

Back Word

LISTENING TO REASON
Closing Remarks



If a tradition has become, implicitly, a dominating force because of the naiveté of our explicit objectification of it in historical study, then . . . a new attitude is called for. This new attitude would recognize the power of tradition for what it really is, treat it accordingly and in that way would attempt to destroy its artificial domination. Heidegger calls this new attitude a "destruction" of tradition. But this "destruction" is positive rather than violently negative. . . . The destruction of a tradition, which he characterizes as a conversation "with that which has been handed down to us," "is not a break with history, nor a repudiation of history, but is an appropriation . . . and transformation . . . of what has been handed down to us. . . . Destruction means—*to open our ears, to make ourselves free from what speaks to us in the tradition as the being of being.*"

—Shaun Gallagher[1]

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[1] Gallagher, *Hermeneutics and Education*, 86. (emphasis added)

Listening to Reason

> It is not merely a question here of confronting ideas but of incarnating them and making them live, and in this respect we cannot know what they are capable of except by trying them out. This attempt involves taking sides in a struggle.
> —Maurice Merleau-Ponty[2]

Some time ago I heard a radio-broadcast on the topic of irradiation as a means of food preservation. As debates go, I found this an interesting one, for it was far more than a mere academic exercise. The participants were passionate about their causes, and the goal of each team was not just to win, but to win over. Each sought to enlighten.

The substance of their arguments was fairly predictable—the affirmative drawing upon scientific evidence; the negative maintaining a more theoretical position founded on the premise that current knowledge might simply be inadequate. And, as also might be predicted, neither side made much headway in convincing the other. As the argument began to heat up, a member of the affirmative (scientific) team finally exclaimed in a fit of exasperation: "If you would only listen to reason!"

As though suddenly aware of the problem, the captain of the negative team responded, out of turn, "No, you're the one who needs to listen to reason."

Listening and Reason

Two interpretations of the phrase "listen to reason" were at play here: the first a demand to *acquiesce to scientific evidence*—in effect, to submit to an external authority; the second a demand to *give serious consideration to the sane and rational argument*—to allow one's inner convictions to be appropriately swayed. Yet, somehow the participants seemed to be neither listening nor reasonable. Both sides missed the import of their mutual invocation. Both sides were demanding the sort of listening that teachers demand of students—a listening that lacks reciprocity, a listening that seeks out precision. Both sides were demanding a reasonableness that stands in a determinate relation to "reality." And so, both senses of listening were passive and submissive; both forms of reason were methodical, monological, and authoritarian.

In the preceding pages, I have attempted to describe a *listening* that is active and generative and a *reason* that moves beyond the constraining bounds of a mathematized rationality. The starting point for my investigation was not that our cultural emphasis on scientific and discursive knowing is wrong, but that, in the words of Morris Berman, it is "pathetically incomplete, and thus winds up projecting a fraudulent reality."[3]

The complicity of mathematics teachers in the modernist project of promoting this fraudulent reality is undeniable. In the classroom we endeavor to convince a captive audience that to be "rational is to be methodical: that is, to have criteria for success laid down in advance."[4] I believe that it is time to broaden our conception of rationality—an

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[2] Maurice Merleau-Ponty, *In Praise of Philosophy* (Evanston, IL: Northwestern University Press, 1963), 27.

[3] Berman, *The Reenchantment of the World*, 270.

[4] Rorty, *Objectivity, Relativism, and Truth: Philosophical Papers, Volume 1*, 36.

evolution that is inextricably bound to a broadening of our conception of listening. Hear Rorty again:

> Another meaning for "rational" is, in fact, available. In this sense, the word means something like "sane" . . . rather than "methodical." It names a set of moral virtues: tolerance, respect for the opinions of those around one, *willingness to listen*, reliance on persuasion over force.[5]

It is a rationality that is incommensurate with current incarnations of school mathematics, but one which need not be incompatible with a mathematics teaching that begins with listening rather than telling.

In the chart below (see Table 3), I have attempted to elaborate on the relationship between our understanding of rationality and our orientation to listening. On the left side is William Irwin Thompson's list of the "four mentalities" through which Western civilizations have passed or are passing. He describes these changes in basic mentality as shifting from one geometry to another.[6] In the right column is David Michael Levin's list of the "four stages" of listening through which the individual passes or might pass on her way to "becoming."

I am suggesting in this comparison that, just as there is a general movement toward "complexification" of our understanding of the universe, so must there be an evolution toward a more complicated (and hence, more difficult) understanding of our relationships with one another. Put differently, the current shift in our cultural knowledge (that is, in our collective patterns of acting) away from the linear static and toward the complex dynamic corresponds to a movement in our conception of the self from the isolated subject/object (the hearer) to the participating and ever-becoming being (the listener). My thesis is that teachers can and must play a role in this development—not primarily by teaching about a new world view, but by enacting it, by listening.

Thompson qualifies his elaboration by adding that most people, thoroughly enframed by modern, linear, and reductive ways of thinking, are incapable of making the sort of radical transformations required to move from the Third Mentality to the Fourth. Levin also points to the constraining effects on listening of the dominant modes of thinking and being. In effect, both authors are acknowledging the slow and uncontrollable processes of change that are implicit in the hermeneutic attitude. As Crusius explains,

> [Hermeneutics] does not maintain that change is impossible or undesirable, only that meaningful sustainable change is gradual, evolutionary, and cannot be effected by critique alone. In short, we can hope to make things better; we cannot hope for wholesale transformation on either the individual or collective level.[7]

This writing is thus not intended to contribute to a badly-needed revolution in mathematics teaching, but to an inevitable evolution. What I hope has been offered here is a possible alternative for mathematics teaching—one which might help to reconnect us to one another, to our knowledge, to nature—but one which I acknowledge cannot be

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[5] Ibid., 37. (emphasis added)

[6] Thompson's framework is quite similar to the five-level analysis that I developed in Chapter 2. A major difference is that his third mentality seems to encompass both my Formalist and my Hyper-Formalist categories. Unfortunately, Thompson does not develop his framework beyond the statements re-presented in Table 3, as it is thus impossible to note further correspondences or differences..

[7] Crusius, *A Teacher's Introduction to Philosophical Hermeneutics*, 72.

extracted from the modernist context of the school. Listening is difficult there, and it is thus that I am comforted by Heidegger's words:

> What we can do in our present case, or anyway can learn, is to listen closely. To learn listening, too, is the common concern of student and teacher. No-one is to be blamed, then, if he is not yet capable of listening.[8]

Table 3. A juxtaposition of a phylogeny of rationality and an ontogeny of listening.

William Irwin Thompson's[9] Four Mentalities of Western Civilizations	**David Michael Levin's[10] Four Stages of Listening**
The first mentality was the arithmetic, the line of counting goods in space and generation of time.	In [the] first phase, our hearing may be said to inhere in, and be attuned by, the field of sonorous Being as a whole . . . The infant's ears are the body as a whole. . . . [It] is an elementary hearing, deeply, symbiotically embedded in the elemental ecology of nature.
The second mentality is the geometric and it expresses the intellectual revolution wrought by Pythagoras and Plato. For these ancients, motion was imperfect and sinful, and only the unmoving geometry of perfect spheres in the ideal realm was a true expression of the Good.	Stage II culminates in a hearing that is personal, adequately skillful in meeting the normal demands of interpersonal living, and ruled over by the ego, which habitually structures all the auditory situations in which it finds itself in terms of subject and object.
The third mentality was the dynamic mentality of modernism, the mentality of Galileo, Newton, Descartes, in which motion and falling bodies became the focus of attention.	[In] the third stage of listening, we are essentially involved in developing our listening as a practice of compassion, increasing our capacity, as listeners, to be aware of, and responsive to, the interrelatedness and commonality of all sonorous beings.
Now we are moving out of this modernist science with its narratives of linear equations into a postmodernist science of which Chaos Dynamics is one important visual expression.	[In stage IV, listening] becomes *a gathering* of sonorous Being: a gathering mindful of its utterly open dimensionality, attentive to the primordial difference by grace of which all auditory structures are possible, and respectful of the incommensuality of the Being of sonorous beings, letting the inaudible be inaudible.

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[8] Martin Heidegger, *What Is Called Thinking?* (New York: Harper and Row, 1968), 25.

[9] Contents of the left column are quoted from William Irwin Thompson, *Imaginary Landscape: Making Worlds of Myth and Science* (New York: St. Martin's Press, 1989), xix.

[10] Contents of the right column are quoted from Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 45-49.

Reason to Listen

As the correspondence between Thompson's and Levin's frameworks suggests, there is great potential in a fuller understanding of the sonorous realm. It is a place that does not permit the simple and lasting distinctions of our visual field. Instead of indisputable see-it-with-my-own-eyes facts, our ears present us with possibilities, complexities, connectings . . . if we are open to interpretations other than our own.

But an openness to other interpretations is only a start to listening. For each action and each word has a history. It carries an echo of humankind's history. The call to listen is thus a call to complexity, to relationality, to transformation, to fluidity. It is a quest for fitness, rather than an adherence to boundaries.

Moreover, words are used by persons, and one's "choice of words" is a reflection of one's personal history, one's context, one's relationships. Each utterance is a chorus of one's dispersed identity—a structure that is "rooted in, and channeled through, the body of our experience. Thus we must not let these processes get cut off from our bodily nature."[11] We must rather listen to our bodies of knowledge, participating in the evolution of those bodies by being mindful of the way we speak.

And so, the call to listen is a call to interrupt what has come to be our "common sense"—that is, of both the prevailing discourse and one's hearing of it. As Heidegger[12] more dramatically states it, we need to develop an attitude of "destruction." It is this attitude that I have attempted to bring to this investigation of mathematics teaching. This sort of positive destruction must be brought to bear against the negative destruction that our society is carrying out under the neutral-sounding labels of "instruction" and "construction."

I have argued that current conceptions of mathematics, of education, of learning, and—perhaps most of all—of teaching are violent, where "violent" is intended to provoke a sense of thoughtless transgression in addition to its more familiar sense of furious destruction. It is a violence that is deaf to (and ultimately silencing of) the voices of its victims—ourselves. Moreover, mathematics teaching is, in my opinion—and I speak here as a teacher who has been complicit in the project—not amoral, as it claims, but indisputably immoral. In allowing itself to forget that its subject matter is a humanity, it has become an inhumanity. It is thus that we have created a system that values compliance over creativity, that spawns destructive behavior by destroying our experience, and that conditions learners to reach for the formulaic ahead of the imaginative.

I left the classroom because I could not abide what I was doing. Although I lacked a means of articulating the source of my dis-ease, I reached a point that I could no longer ignore it. I think that I have now found a language to express those troubling intuitions, a language to support alternative patterns of acting, and a language that announces the sort of transformations that I have undergone through this project.

Has it been a success? I cannot be the final judge. In the end, this research was worth the effort *not* if it has convinced everyone to listen to reason, but if it has provided someone with a reason to listen.

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[11] Levin, *The Listening Self: Personal Growth, Social Change and the Closure of Metaphysics*, 174.

[12] Heidegger, *What is Philosophy?* (See the epigram on the title page of this section.)

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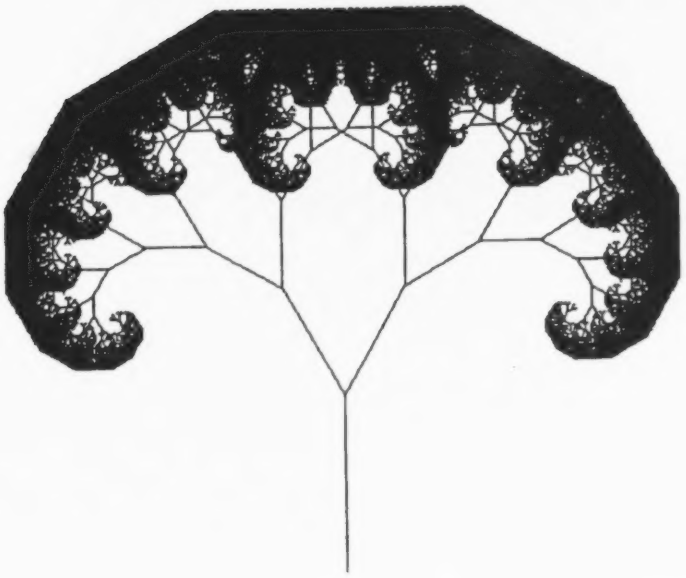

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